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# Experimental evidence of density fluctuations in two-dimensional bins

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**Abstract**

We have found experimentally the existence of random density fluctuations, during the discharge of monodisperse granular material from flat bins, which follows a $1/f^{\alpha}$ noise, where $\alpha \simeq 1.32$ just above, and $\alpha \simeq 1.0$ just below the exit of the bin. The corresponding rescaled range analysis for the fractal noise obtained just above the exit gives a Hurst parameter $H \simeq 0.16$, and a fractal dimension $D \simeq 1.84$.

*Keywords:* Mechanics of discrete systems; Non-Newtonian dynamics; Mechanical and acoustical properties of condensed matter

In the study of granular media, like sand, fluctuating phenomena have been recently noted under several conditions of composition and motion. A special class of these phenomena seems to contain $1/f^{\alpha}$ noise, which is a signal of random behavior as well as of a lack of characteristic time scales [1-11]. Phenomena obeying noise with these characteristics have been detected during the surface flow in small sandpiles [1], in the sound propagation inside a granular material [2,3], in the flow through long narrow pipes [4-9] and in the stress on the side wall during the outflow in a three-dimensional (3D) conical hopper [10,11], among others. Perhaps the main possible cause of these noisy behaviors can be the inherent internal disorder of the granular material which can respond with strong fluctuations to small induced changes.

In a recent experiment of the outflow of granular material from flat hoppers filled with dry granular material, like common building sand, the formation of patterns was observed [12,13], which seems to indicate qualitatively the existence of noisy density patterns. In despite of density patterns were noted in this experiment [12] and another more idealized experiment [14], to our knowledge the corresponding experimental time scale analysis has not been previously reported.

More recently, Ristow et al. [15], using molecular dynamics simulations (MDS), have characterized the temporal fluctuations associated to the density patterns, during the discharge of 2D hoppers, as obeying a $1/f^{\alpha}$ noise, where $\alpha = 2.7 \pm 0.2$. In the present work we report experiments in nearly 2D vertical bins (flat bins) which let us establish the existence of $1/f^{\alpha}$ noise with a strong different value of $\alpha$ compared with that obtained from simulations [15]. Like in the case

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of the dynamic stress measurements within conical hoppers [10,11], we also have used Hurst's rescaled range analysis ($R/S$ analysis) to detect the possible fractional Brownian motion [16,17] for the density patterns.

So, the purpose of this work is twofold. First, we search the noisy character of the time series of the number density at two positions just above the exit (within the bin) and just below the exit (out of the bin), respectively, of flat bins during a gravity induced flow of monodisperse granular material. Second, we look for the existence of fractional Brownian motion because the corresponding power spectra for the time series of the number density within a bin show a $1/f^{\alpha}$ noise, with $\alpha \simeq 1.32$.

In order to observe and characterize the density fluctuations we did experiments in vertical glass-bins with the following dimensions: 40 cm length, 40 cm height and 0.4 cm width. The bins were filled by raining down the grains up to $h = 36$ cm. The granular material consists of monodisperse rough glass beads with a mean grain size $d_0 = 0.315 \pm 0.004$ cm in diameter, grain density $\rho_p = 2.45$ gr/cm$^3$, and a friction coefficient $\mu = 0.57$ in the Coulomb sense. The size of the aperture, $d_a = 2$, cm was maintained fixed. It is to be noted that the ratio thickness of the bin to the particle diameter is close to 1.27, thus assuring a near 2D flow. The bottom walls form a channel at the exit of length 4 cm by width 2 cm.

Our measuring technique uses light produced by an halogen lamp, passing through the granular material at time $t$, which is captured by a silicon solar cell, type Archer 276-124A, situated in front of the bin (Fig. 1). The photocell, positioned at 0.6 cm from the bin wall, has a cross-sectional area 2 cm × 4 cm and, therefore, the actual measurement cross-sectional area was almost the same as that of the photocell. The light variations produced continuously by the granular flow were monitored in real time by the photocell and the voltage variations were transmitted to a PC using an A/D converter coupled to a data acquisition card PCL-818. With this method we resolve even a fraction of a single grain.

We have searched the effect of the exit locations on the number density. This was motived by other authors [18–20] who detected a noticeable increase in the mass flow rate as the exit is moved from the center to the border. Thus, we have done measurements

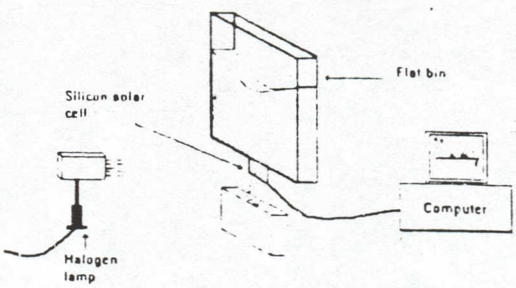


Fig. 1. Schematic lateral view of a flat bin. The bottom exit was placed in two different positions: (I) at the center and, (II) at the border. In both cases we have measured the number density in the vicinity just above the exit of the bin and just below the exit.

of the time-dependent number density using two different configurations: (I) the exit located at the center and (II) the exit located at the border. Also, the photocell was placed in two different relative locations: (1) just above the exit (to detect the number density variations within the bin) and (2) just below the exit (to detect the number density variations out of the bin). In both cases (I) and (II), the voltage measurements were made each $\Delta t \simeq 1.16 \times 10^{-3}$ s. The measurements showed considerable fluctuations in the voltage which corresponds, after calibrations, to considerable fluctuations in the number density. A full account of our work, including a detailed discussion of the instrumental noise in the experiments, will be published elsewhere [21].

Depending on the photocell position we found two different calibrations relating the measured voltage $V$ with the number of grains in the cross-sectional area. The non-linear calibration formulas are

$$V(t) - V_0 = k[n^2(t) - n_0^2] \quad \text{for cases (1)}, \qquad (1)$$

and

$$V(t) - V_0 = kn^2(t) \quad \text{for cases (2)}, \qquad (2)$$

where $k = -1.6 \times 10^{-5}$ V and $V_0$ is the initial voltage without flow, $n_0$ is the initial number of grains in a cross-sectional area with the same dimensions as that of the photocell, and $n(t)$ is the number of grains at the time $t$. Clearly, in Eq. (2) there is not an initial number of grains because the solar cell is placed just below the exit, while in Eq. (1) there is an initial number of grains because the solar cell detects light passing a zone within the bin. By using Eqs. (1) or (2)

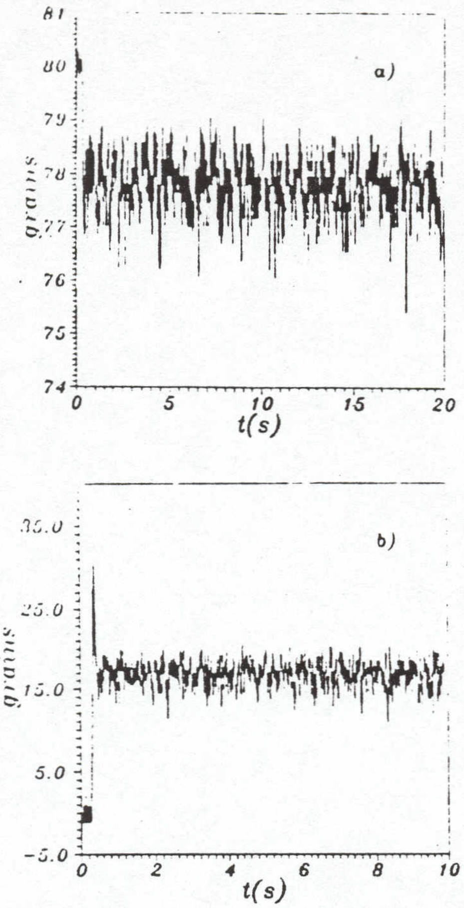


Fig. 2. Typical noisy time series of the number of grains in a 2 cm × 4 cm cross-sectional area (a) just above the exit and (b) just below the exit. Initially, within the bin, there is a certain quantity, $n_0$, of grains while, just at the exit, $n_0 = 0$; the small fluctuations around the mean zero are due to the measurement technique. The time series in (a) has a larger duration than that in (b) because the first series is typical of measurements with exit at the center while the second series is typical of measurements with exit at the border, i.e., this last one corresponds to the larger mass flow rate.

we performed a calculation of the number of grains in a 2 cm × 4 cm cross-sectional area perpendicular to the flow, as a function of time, within the bin (Fig. 2a) and for comparison, outside the bin (Fig. 2b). We have used $n_0 = 80$ in Fig. 2a, which was obtained from experiments, and $n = 0$ in Fig. 2b. The small fluctuations around $n = 0$ are due to the noise produced by the measuring technique.

We have performed a fast Fourier transform (FFT) analysis for the time series which let us obtain the power spectra (and the spectral logarithmic slope $\alpha$) and, therefore, the possible characteristic frequencies associated to the time series of the number of grains for both cases (I) and (II) and for both cases (1) and (2). In experiments ten trials have been studied in each of the four cases. We have found different averaged power spectra between cases (1) and (2) but not between cases (I) and (II), i.e., the exit position was not important in relation with the approximate value of the $\alpha$ exponents.

In cases (1) the averaged power spectra of the number of grains, $n(t)$, goes like $1/f^{\alpha}$ with $\alpha \simeq 1.32 \simeq 4/3$ over one decade in the range of frequency $f$ (Fig. 3a) which is in disagreement with the predicted spectrum by Ristow et al. [15] for 2$D$ hoppers. They obtained an overestimate of the exponent $\alpha$ compared with that obtained in our experiments. In cases (2) the averaged power spectra of $n(t)$ shows an exponent $\alpha \simeq 1.0$ over more than one decade in the frequency range, this latter case corresponding formally the ubiquitous power spectra $1/f$ (Fig. 3b). All the $\alpha$ exponents reported here were obtained using a least squares fit over each averaged spectrum which was computed after the time series, $n(t)$, was divided into 16 segments; for each case treated here the values of $\alpha$ have an error of less than 3%.

Motivated by the above results, we then examined the existence of fractal noise. The random process characterized by Brownian motion yields a power law spectrum with $\alpha = 2$. Our data for the number density within the bin (cases (1)), with $\alpha$ near but typically larger than 1, suggest the possibility of fractional Brownian motion (fBm) [16,17]. Such a possibility for the series may be found using $R/S$ analysis which we outline briefly here. In this work we obtained records in time of the number density variations. By choosing a time period (referred to as a lag) shorter than the total record, we first calculate the increments $I = n(t+\Delta t) - n(t)$, the change in the signal between two adjacent times for this lag, and then the time average $A = \langle I \rangle_{\text{lag}}$, and the standard deviation $S$ of the increments. By constructing a set of new increments $(I - A)$, where

$$(I - A) = \{(n(t+\Delta t) - n(t) - A),\; (n(t+2\Delta t) - n(t+\Delta t) - A), \ldots,\; (n(\text{lag}) - n(\text{lag}-\Delta t) - A)\},$$

and sequentially adding them, we reconstruct an image

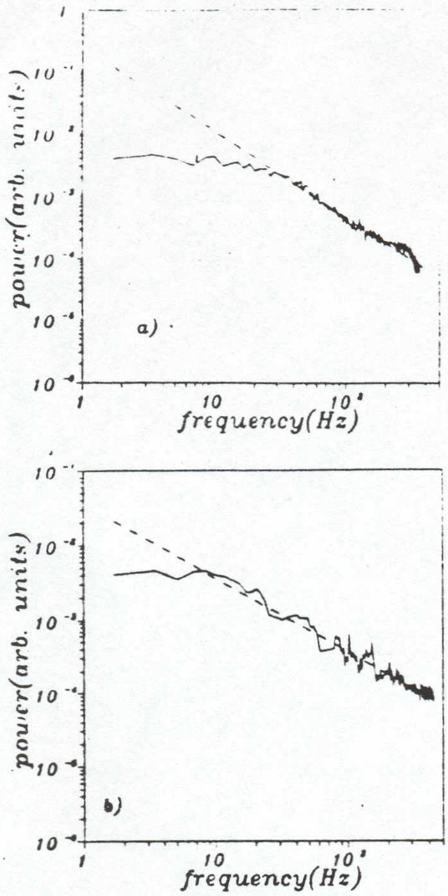

Fig. 3. Log-log plot of the averaged spectra corresponding to the full frequency range available from a typical experiment for the two cases here studied: (a) within the bin and (b) at the exit of the bottom opening. In cases (1) $\alpha \simeq 1.32$, and in cases (2) $\alpha \simeq 1.0$. The corresponding exponents were obtained by using a least squares fit for the data over each averaged spectrum. The averaged spectrum was produced by partitioning the time series into 16 overlapping increments and, after, averaging over the 16 individual spectra.

of the original curve in this lag along the horizontal axis. The range $R$ is defined as the maximum value minus the minimum value of the curve, and by dividing $R$ with $S$ we obtain a dimensionless number $R/S$ for the lag in question. If the lag was chosen say 1/100 of the total period, we have 100 independent estimates of the $R/S$ for this lag, and the final value is the average of all of them.

For many natural phenomena it has been found that the number $R/S$ has the form

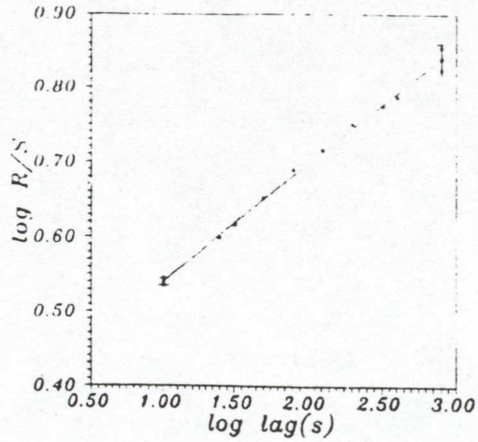

Fig. 4. The $R/S$ analysis for the time series of Fig. 2a for both exit located at the center and exit located at the border. The line which adjust the points has a slope $H \simeq 0.16$ for both cases.

$$R/S \sim (\text{lag})^H, \quad (3)$$

where $0 < H < 1$, and $H$ is now referred to as the scaling exponent or Hurst parameter. Classical random walks (Brownian motion) correspond to $H = 0.5$ [16]. If $H > 0.5$, then past and future increments are positively correlated, whereas if $H < 0.5$, then past and future increments are negatively correlated [17]. Also, it is easy to prove that the $\alpha$ coefficient of the spectra and the Hurst parameter can be related by $\alpha = 2H + 1$ [17]. Finally, the fractal dimension $D$, a qualitative measure of its filling properties [16], can be expressed in terms of $H$ as $D = 2 - H$.

Therefore, we can obtain the Hurst parameter for the data obtained in cases (1) by using the $R/S$ analysis or by using the relation $\alpha = 2H + 1$ with $\alpha \simeq 1.32$; both approaches give a scaling exponent $H \simeq 0.16$ and, therefore, we obtain a fractal dimension $D \simeq 1.84$.

In Fig. 4 we show the $R/S$ analysis for cases (1). It is insensitive to the exit position. This result indicates that the time series in cases (1) have fractal noise and are negatively correlated random walks. The fit in Fig. 4 has been made over less than one decade in the $R/S$ range, because the number density fluctuations are much smaller than the mean value.

Giving these findings, we are unable to reject the existence of noise associated to the number density in monodisperse material. The simulations of Ristow et al. [15] which gives a $1/f^{\alpha}$ noise, where $\alpha = 2.7 \pm 0.2$, can also show the existence of this type of noise for monodisperse granular material; however, it is clear that the MDS do not reproduce our results for cases (1) ($\alpha \simeq 1.32 \simeq 4/3$) overestimating strongly the value of the exponent $\alpha$. The presence of a $1/f$ noise just below the exit of the 2D bins was neither investigated in the MDS.

In Figs. 3a and 3b we show the existence of cutoffs in the power law of the power spectra, which has not been observed using MDS. The existence of low and high frequency cutoffs in the power spectra ensure, actually, that the total power does not diverge to infinity [22].

We will give a more detailed physical discussion of the power spectra and their frequency cutoffs for all cases treated here. The lower frequency cutoff is due to the finite length $T_{max}$ of the time series and is equal to $f_{lower} = 1/T_{max}$; typically in our experiments $f_{lower} \sim 10^{-3}$ Hz for cases (II) and $f_{lower} \sim 10^{-2}$ Hz for cases (I). The high frequency cutoff is due to sampling, and is equal to the Nyquist frequency $f_N = 1/2\Delta t \simeq 400$ Hz. An intermediate frequency, the frequency of transition $f_{int}$, which separates the high frequency and the low frequency regions for the power spectra can be determined, in cases (1), from the residence time of a grain through the vertical length $l_r = 4$ cm, of the photocell window which is just at the exit of the bin. In this case, because the grains fall down freely under the gravity acceleration $g$, the characteristic residence time $t_r$ is of the order $t_r \sim \sqrt{2l_r/g} \simeq 0.1$ s and then $f_{int} \sim 10$ Hz. Characteristic times larger than $t_r$ imply lower averaged velocities and therefore clustering occurs. So, signals above $f_{int}$ describe clustering obeying a $1/f$ noise while signals captured at a frequency below of $f_{int}$ are $\delta$-correlated. In our experiments there exists also an intermediate frequency in cases (2) and similar consequences as given above follow for these cases.

In order to obtain a more realistic value of the exponent $\alpha$ for the power spectrum within the bin, we suggest to introduce the shear friction force in MDS proportional to the square of the relative velocity of the grains. We have recently shown that this form gives a good approximation to the dissipative stresses in a rapid granular flow within a bin [23].

In summary, we have presented evidence for the complex noisy behavior of the density patterns during the discharge of a 2D bin. From a physical point of view the existence of $1/f^{\alpha}$ noise lets us confirm the lack of characteristic time scales during the gravity induced flow from flat bins and the existence of long time correlations decaying exponentially. We should again emphasize that the position of the exit was not important in relation with the approximate values of these exponents, which seems to indicate that the asymmetry in the value of the mass flow rate noted by other authors [18-20] is not related with the local density variations. More detailed studies not yet made, such as the spatial correlation between density patterns, flow behavior, collective properties of this flow and stability against random fluctuations, among others, are needed in order to reach a better understanding of the nonlinear dynamics of these systems.

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