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Experimental study of the tracer in the granular flow in a 2D silo

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Abstract

We present an experimental study of the dispersion of tracer particles close to the center, during the gravity flow of monodisperse granular material in a flat silo. We obtained the mean and the mean-square displacement of the tracer position as a function of time. The local velocity along the vertical direction shows a clear dependence on the height, which is due to the shape of the silo and the material friction properties. © 1998 Published by Elsevier Science B.V.

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1. Introduction

The flow of granular material from storage devices such as hoppers and silos, obeys a quite complex dynamics which can produce strong fluctuations in the pressure field and in the density during the discharge [1–3], among other related phenomena. The origin of these fluctuations lies necessarily in the motion of single grains and, thus, is very important in obtaining a description of the dynamics at this level. Therefore, in order to characterize the local motion of a single grain, we have made an experimental study of the trajectory of tracer grains with several initial positions, for the gravity induced granular flow in a two-dimensional (2D) flat-bottomed silo, with the exit hole located at the center. The main differences with a similar approach made in Ref. [4] is that we study the flow in a silo (instead a channel) and we follow the track of a single particle (Lagrangian frame of reference) instead of employing the Eulerian frame of reference.

In general, the granular material within a silo has a high bulk density. Due to the strong interactions between particles, the motion of a single grain has important temporal and spatial fluctuations. The grains above the stagnant zone (above the stagnant angle) tend towards the bottom exit under shocks and long-duration frictional contacts. This situation produces, in this zone, an overall bulk motion, which induces in single grains a dispersive behavior.

This experimental study can test the validity of computer simulations, such as the distinct element method (DEM), to study the flow of the enormous number of individual particles and their effect on a single grain (tracer) close the center. These codes use Newton's laws of motion by taking into account the normal and tangential interactions [5]. We experimentally obtained in this work the motion of single tracers with a zero averaged transversal velocity. We have found a

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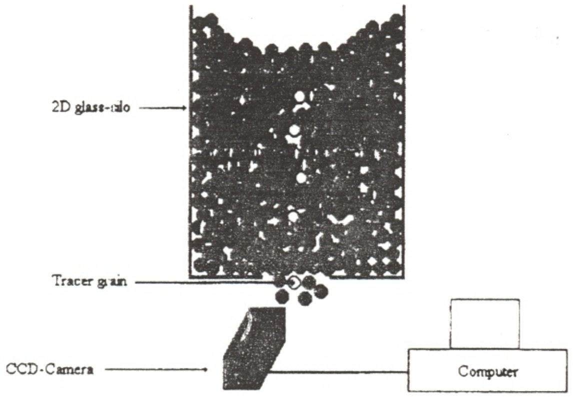

Fig. 1. Schematic picture of the experimental setup. The non-overlapping bead arrangement indicates the 2D structure of the granular system. White glass-beads are the tracers.

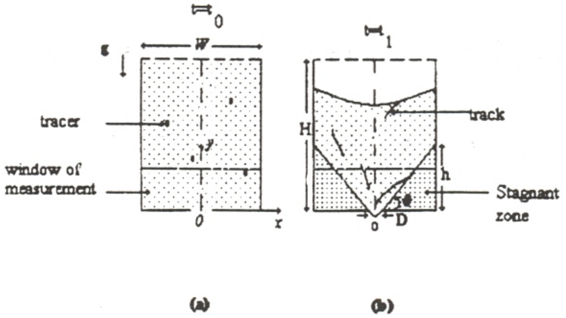

Fig. 2. Geometrical configuration of our silo filled up to a height $H = 80$ cm and with a width $W = 30.8$ cm. (a) Initial arbitrary distribution of grains. The size of the tracers is exaggerated. At the bottom we show the window of measurement and the coordinate system. (b) Tracks during the granular flow. The stagnant zone has a height h, an angle of approach θ and an exit size D. The free fall arch has a radius R_0.

regular agreement between our experimental results, for the vertical velocity as a function of time, and those obtained by using DEM codes. The outline of this Letter is as follows. In Section 2 we describe the experimental setup and the measurement procedure. We present in Section 3 the data analysis and the main results for the mean-square displacement and for the velocity of tracers near the center (axis of symmetry) of the silo. Finally, in Section 4 we give a discussion of the results and present the main conclusions.

2. Experimental set-up

In order to examine the particle trajectories, we have used a vertical glass-silo of 100 cm height, 30.8 cm width and 3.8 mm deep. This silo was filled up to $H = 80$ cm with monodisperse granular material composed of spherical glass-beads with mean diameter $d_p = 3.15 \pm 0.04$ mm, mean friction coefficient $\mu = 0.57$ and grain density $\rho_p = 2.45$ gr/cm^3. Fig. 1 shows the schematics of the experimental setup. The ratio of the thickness of the silo to the particle diameter is close to 1.2 assuring a monolayer of spherical grains. The tracers were grains of granular material colored with fluorescent dye; its motion being recorded by illuminating the silo with black light, obtaining a high spatial resolution. In order to follow with detail the motion of the tracers, the recording and measurements were made in a window of 22 cm height by 30.8 cm width. The bottom of this window coincided with the bottom of the silo (Fig. 2a). The silo had a bottom exit with a size $D = 2$ cm, located just at the center.

We followed tracers with initial heights $h_1 \simeq 12.5$ cm, $h_2 \simeq 20.5$ cm, $h_3 \simeq 29$ cm and $h_4 \simeq 40$ cm, measured from the bottom of the silo. With these initial heights we assure that the tracers always follow the main flow. At time $t = 0$, we have opened the bottom exit and the lower part of the granular material was immediately moved. This motion induced by gravity was recorded using a CCD camera and the noisy paths of the tracers have been analyzed in a computer. The subsequent time series of the other tracers were always referred with respect to the initial time of the lower tracer. The position versus time information needed to determine the transversal and longitudinal components of the local velocities of the tracer particles was obtained by putting the origin of the coordinate system at the middle of the lower edge of the window (Fig. 2b). The tracer particle paths were recorded with a fast-shutter-speed (1/100 s) video system with a recording frequency of 30 frames per second. A dozen of trajectories (tracks) were recorded for each tracer grain with an initial position (x_0, y_0). The time elapsed for a particle to reach the exit was typically between 2 and 5 s (i.e.. 60–150 frames with the sampling period of the time series, $\Delta t = 1/30$ s).

A typical trajectory of the tracer during the flow is given in the Fig. 3a. Here x indicates the transversal component and y indicates the longitudinal compo-

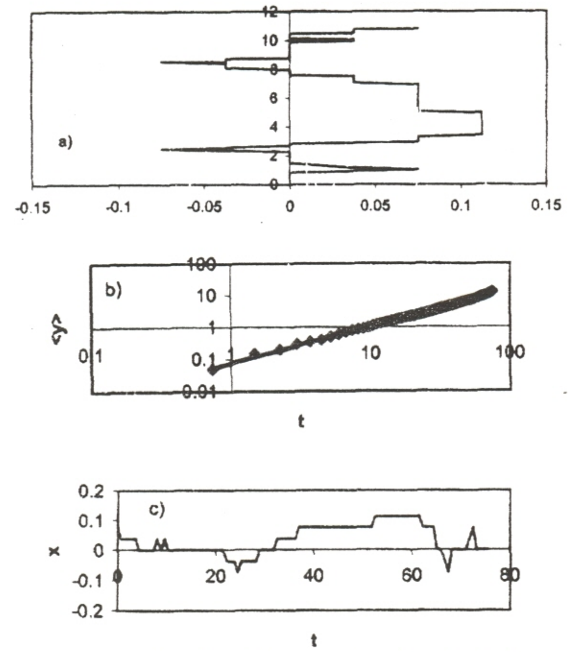

Fig. 3. (a) Non-dimensional characteristic trajectories for the tracer particles during the discharge. All lengths were scaled by D and the time by $\sqrt{D/g}$. (b) Log–log plot the typical behavior of the longitudinal component as a function of time. (c) Typical noisy behavior of the transversal component as a function of time.

nent. The time dependence of each spatial component was analyzed in all cases. In Fig. 3b we show the typical behavior of the longitudinal component as a function of time, t, and in Fig. 3c we show the noisy behavior of the transversal component. We note that for the longitudinal component, a non-linear relationship exists between the coordinate y (or between its time average $\langle y \rangle$) and t, to be analyzed in the next section. The deviation with respect to the linear behavior suggests that the longitudinal mean-square displacement (msd) and the longitudinal average velocity must obey a more complex dependence on t and consequently a very rich dynamics. In the next section we present a discussion of these issues and their importance in relation to the dispersive nature of the tracer motion.

3. Experimental results

In order to show experimental evidence of dispersion (fluctuations of the velocity around the main flow), we have analyzed the average and mean-square displacements for all trajectories as a function of time. We also have computed the local velocities for each tracer, finding a regular agreement between our experimental results and the computer simulations obtained in a similar flow configuration [5]. A very important result from the present study is that identical results were obtained by studying the longitudinal component of the trajectories for tracer particles a few grain diameters off the center.

3.1. *Dispersion of grains*

For the following discussion we introduce non-dimensional variables, where the lengths, x and y, are scaled with the exit size, D, and the time t, by $\sqrt{D/g}$. The ratio of the exit size to the grain size in our case is $D/d_p \simeq 6.35$. In Fig. 3b we show the typical behavior of the longitudinal component, $y(t)$, for a tracer close to the center of the silo (aligned with the exit). The time average of $y(t)$, $\langle y(t) \rangle$, can be well correlated with

$$\langle y(t) \rangle \simeq a t^{\beta}, \tag{1}$$

where a and β are constants. The exponent β has been calculated after an average for all values of β (each one of those obtained by using a least-squares fit over the corresponding time period). This method gives $\beta \simeq 1.16 \pm 0.07$, where the error was of a random nature. This result suggests immediately that the motion along the longitudinal component is slightly accelerated. On the other hand, in the measurements with tracers close to the center, we obviously obtain $\langle x \rangle = \langle v_x \rangle = 0$.

Due to the existence of a main flow along the longitudinal direction, the mean-square displacement was computed for several trajectories of tracer particles, by using the time average $\langle [y(t) - \langle y(t) \rangle]^2 \rangle$. This also can be correlated by

$$\langle [y(t) - \langle y(t) \rangle]^2 \rangle \simeq b t^{\gamma}, \tag{2}$$

where b is a constant and the average value of γ is $\gamma \simeq 2.23 \pm 0.13$. See Fig. 4a. This value of γ being obtained in a similar way as that of β. A better representation of the displacement fluctuations is obtained by normalizing this quantity with $\langle y(t) \rangle$. In this case

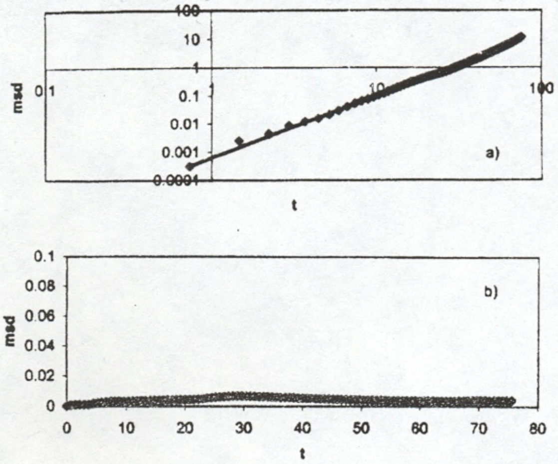

Fig. 4. (a) Log–log plot of the non dimensional mean-square displacement (msd) as a function of time for the longitudinal displacement of the tracer particle. (b) Mean-square displacement for the transversal displacement.

$$\frac{\sqrt{\langle [y(t)-\langle y(t)\rangle]^2\rangle}}{\langle y(t)\rangle} \sim t^{\gamma/2-\beta} \sim t^{-0.035\pm0.005}. \qquad (3)$$

The rms of the fluctuations related to its mean value seems to decrease then with time. The values of β, γ and $\gamma/2-\beta$ are strictly valid in the window of measurement and do not depend on the initial position of the tracer.

The power-law behavior in relation (2) indicates that, near the bottom, the motion of a single grain in the flow is neither ballistic (where $\gamma = 2$) nor uniformly accelerated (where $\gamma = 4$). A small damping occurs, which is not so strong as in case of percolation, of a small grain in a monodisperse, three-dimensional, static granular medium under the gravity action. There, a diffusive behavior occurs along the transversal direction [9]. This last phenomenon occurs because the motion of the moving particle is dominated by shocks and consequently gravity is balanced by an effective friction due to the medium.

For the transversal component, the diffusion for tracers close to the central axis is negligible because

$$\langle [x(t)]^2\rangle = 0. \qquad (4)$$

This result being showing graphically in Fig. 4b. For tracer grains a few dozens of grain diameters off the center, the mean-square displacement is neither diffusive nor superdiffusive [6–8] and a more detailed analysis of this case should be necessary.

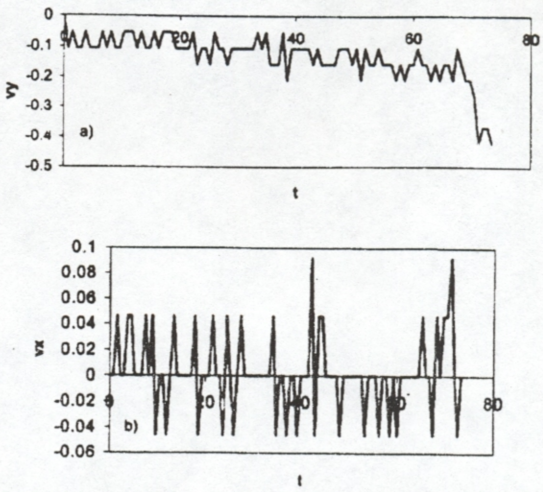

Fig. 5. (a) Typical behavior for the non dimensional vertical velocity, v_y, as a function of time. (b) Typical behavior for non-dimensional transversal velocity, v_x. Here we used the scaling $v_y \rightarrow v_y/\sqrt{gD}$ and $v_x \rightarrow v_x/\sqrt{gD}$.

3.2. Local velocities

The local longitudinal velocities of the tracers were evaluated in order to remark on similarities or differences with respect to the computer simulations, where the detailed interaction between grains was taking into account [5]. In Fig. 5a we show a typical plot of the local longitudinal velocity, v_y, as a function of time. For comparison, in Fig. 5b we show the behavior of the transversal velocity component, having a noisy structure with an averaged value equal to zero. Qualitatively, the graph of the local longitudinal velocity component has a similar structure to that shown in Ref. [5]. However, a more detailed quantitative analysis shows a more complex behavior. Our main result in this sense is that the averaged longitudinal velocity component increases smoothly with time, increasing abruptly close to the bottom exit.

This abrupt change is shown in Fig. 5a, but also it can be deduced in terms of the dependence of v_y with y. In Fig. 6 we show the averaged behavior of v_y as a function of y. From this figure we note that, depending on the value of the y coordinate, there are at least two zones with different motional regimes for the longitudinal velocity component in the measurement win-

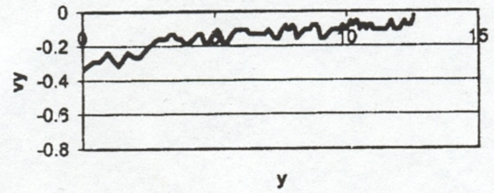

Fig. 6. Average behavior of the vertical velocity, v_y, as a function of the longitudinal component, y.

dow: A zone, between $3.3 \leqslant y \leqslant y_l$, where the flow takes place in a converging channel, due to the presence of the stagnant zone (See Fig. 2b). Because the averaged mass flowing towards the exit is conserved, the vertical length of this zone has the dimensionless value

$$y_l \simeq h^* = [(W^* - 1)\tan\theta]/2 = 11.3, \tag{5}$$

where $h^* = h/D$ is the non-dimensional height and $W^* = W/D$ is the non-dimensional width. The angle θ, called the angle of approach [10], is a function of the container shape and of the material. In our experiments we found that $\theta \simeq 1$ (57.5°). In this converging zone, the time average of the longitudinal velocity can be well correlated by

$$\langle v_y \rangle \simeq c t^{\epsilon}, \tag{6}$$

where ϵ has, after an averaging, the value $\epsilon = 0.331 \pm 0.02$ and c is a constant.

Fig. 6 also indicates the existence of another zone of different granular motion close to the exit. In this zone, with $R_0 < y < 3.3$, we have found that $\langle v_y \rangle$ changes suddenly to a linear dependence with respect to time. Here, R_0 is usually called the radius of the free fall arch (see Fig. 2b), so that $R_0 \simeq 1$. Below this zone the particle increases its velocity as a free particle under the gravity action. These results show that the motion of the tracer changes qualitatively at a distance of three times the exit size above the flat bottom.

Finally, above $y = y_l$ (out of our window of measurement), there is another zone of granular motion. In this upper zone we have measured the longitudinal velocity component by using a two-frame cross-correlation technique, similar to that employed in the particle image velocimetry (PIV) [11]. We found that its value is approximately constant with a non-dimensional value $\langle v_y \rangle \simeq 0.04$.

4. Discussion and conclusions

The most interesting case studied here was the corresponding to the motion of tracer grains close to the center and close to the bottom of a quasi-2D silo. Cases where the tracers are initially far from the center are more complicated. We found that the relative intensity of the displacement fluctuations in the longitudinal direction seems to decrease as the tracer particles approached to the exit in the middle or converging flow zone. In the transversal direction, the motion is found to be neither diffusive nor superdiffusive, depending mainly on the distance from the center. This anisotropy also is noted in the local velocities along the transversal and longitudinal directions.

The geometry of the silo and the friction of the material also affects the longitudinal velocity due to the presence of the stagnant zone which induces a converging flow. We have detected in the silo three different zones where the velocity along the longitudinal direction has a qualitatively different dependence on time. In the upper zone, $y > h^*$, the average velocity is approximately constant. In the converging zone ($3.3 \leqslant y \leqslant h^*$) the average velocity changes as $t^{0.331}$, and close to the exit, $y \leqslant 3.3$, the average velocity changes linearly with time. This behavior of the longitudinal velocity is also maintained for tracer grains a few grains off-center of the silo. Therefore, these results show that the behavior of the velocity, as a function of the position, is actually more complex than that found in DEM simulations for the 2D silo. The transversal velocity should, certainly, change in an important way along this direction, even if the distance between two grains is only a few grain diameters. This high anisotropy in the velocity and in the displacements was previously detected experimentally in a 2D channel [4] and seems to be due to the very important variations of the shear stresses. However, an experimental study of these shear stress variations has not been yet made. The flow case, where the grains tend towards the center of the silo from a distance many times the grain diameter, is also very interesting and work along these lines is in progress.

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