Available online at www.sciencedirect.com
SCIENCE DIRECT®
PHYSICS LETTERS A
Physics Letters A 324 (2004) 14–21
www.elsevier.com/locate/pla

Imbibition in a Hele–Shaw cell under a temperature gradient

M. Sanchez [a], F. Sanchez [a], C. Pérez-Rosales [a], A. Medina [b,*], C. Treviño [c]

[a] *Grupo de Medios Porosos y Granulados, Programa de YNF, IMP, 07730 México D.F., Mexico*
[b] *ETSI Aeronáuticos, Universidad Politécnica de Madrid, 28040 Madrid, Spain*
[c] *Facultad de Ciencias, UNAM, 04510 México D.F., Mexico*

Received 12 May 2003; accepted 26 January 2004

Communicated by A.R. Bishop

Abstract

Imbibition in a Hele–Shaw cell under a constant, longitudinal temperature gradient G was analyzed both theoretically and experimentally. A theoretical approach based on a motion equation was developed in this case and a closed-form analytical solution has been obtained. Theory shows that the time evolution of the front is strongly dependent on the way in which the surface tension and the viscosity change as a function of the temperature. Experiments using parallel glass plates and glycerol were made under several gradients and they have fitted well to theoretical profiles.

PACS: 44.10.+i; 47.80.+v

1. Introduction

The regime of motion of a liquid free surface can be affected by the presence of changes in the surface tension, σ. A simple way to change the surface tension is by changing the temperature T on a free surface because, in general, in liquids the surface tension decreases with increasing liquid temperature, i.e., $(\partial\sigma/\partial T) < 0$. The presence of this type of variations leads to the appearance of tangential stresses on the liquid surface and consequently to thermocapillary flows [1,2]. Some representative phenomena related to this type of flows are the film flow developed on a vertical plate, having a negative temperature gradient, when it is partially submerged into a liquid reservoir [1–7], the flotation of a liquid drop on a liquid surface when the surface has different temperature than that of the drop [8] and the motion of liquid bridges due to non-isothermal conditions [9].

Temperature variations can also produce changes in other type of flow driven by surface-tension known as imbibition, which is the spontaneous penetration of a liquid into a capillary due to the action of the driven capillary force $F_c = p_c S$, where p_c is the capillary pressure and S is the area of the free surface [1,10–16]. When imbibition occurs in a vertical capillary, the capillary force dominates over the gravity and frictional forces. This mechanism should be valid even under the presence of temperature gradients on the capillary

* Corresponding author. On sabbatical leave from IMP, Mexico.
E-mail address: amedina@imp.mx (A. Medina).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2004.01.070

walls but in this case the dynamic viscosity and surface tension are temperature-dependent quantities. So, the main aim of this work is to present a study of imbibition in a Hele–Shaw cell when a longitudinal, constant temperature gradient is considered. The main motivation of this work is that imbibition occurring in porous media is a complex process; however, it is of enormous importance, for instance, in the exploitation of heavy crude oil from subterranean fractured reservoirs where large volumes of oil have been recovered from several supergiant reservoirs [17]. In the fractured reservoirs the host rock can be assumed as impervious and the fluid flows occur along the very slim, fluid-filled fractures. In subterranean reservoirs the geothermal gradient is ubiquitous and it has a value approximately constant [18] but, additionally, the reservoir production can be tailored when heat is introduced into the reservoir by injecting steam or hot water which creates dynamic liquid–gas interfaces under the presence of strong temperature gradients close the injection zones [19,20]. Owing to all these previous considerations a special type of imbibition can be seen as occurring in Hele–Shaw-like cells under temperature gradients. Despite the frequency and importance of this type of imbibition a physical understanding of this problem is absent to date.

To achieve a better comprehension of imbibition into vertical fractures, a theoretical and experimental study for imbibition in Hele–Shaw cells under constant temperature gradients is presented in the following sections. In Section 2 the problem is formulated for a one-dimensional flow where linear laws for the temperature-dependence of dynamic viscosity and surface tension are assumed. The resulting motion equation yields a closed-form analytical solution for the evolution of the averaged imbibition front as a time function. Clearly, this solution has an explicit dependence on temperature gradient and, as an asymptotic limit, the Washburn law [10] valid for isotherm imbibition and small distances of advance of the front, will be held. Afterwards, in Section 3, experiments in Hele–Shaw cells made of two parallel glass plates and having several values of temperature gradients are carry out using glycerol. Good agreement between experimental data and theory shows the validity of the present approach. Finally, in Section 4, the conclusions and main limitations of this study are given.

2. Theoretical approach

In Fig. 1 a schematic view of imbibition in a Hele–Shaw cell is shown. As soon as the lower part of a couple of parallel plates (making a gap of thickness D, length L and height H) touch a liquid reservoir, a capillary flow is developed inside the cell under the action of the gravity acceleration, g. After an elapsed time t the imbibition front reaches the average distance $h(t)$. Normally, studies of this process have been made by assuming that the overall cell and the liquid have a constant temperature $T = T_0$ and, consequently, the flow is developed under isothermal conditions [16].

Here it is considered that the Hele–Shaw cell has a longitudinal, constant temperature gradient $G = \Delta T/H$, where $\Delta T = (T_1 - T_0)$ is the temperature difference between the upper (T_1) and lower (T_0) parts of the cell, as shown in Fig. 1. Thus, the capillary penetration will be treated by considering that during its advance the fluid (of thermal diffusivity α_f) reaches the same temperature distribution as that of the solid. This last condition is met, for instance, if the diffusive time $t_D = D^2/\alpha_f$ is very small compared with the imbibition transit time, $t_I = D/|dh/dt|$. Clearly, this is the case when the plates are very close

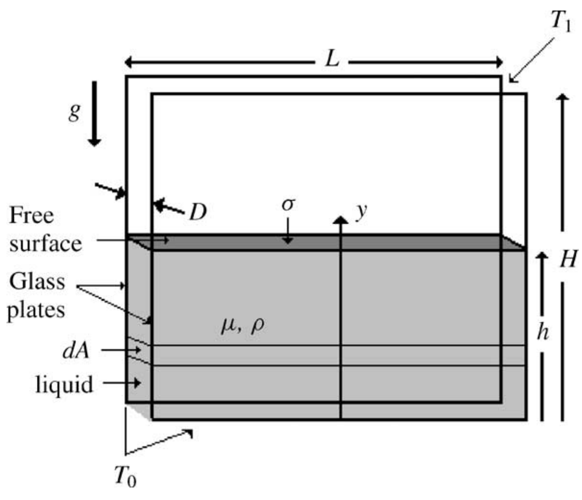

Fig. 1. Schematic view of the geometrical and physical parameters during the imbibition in a Hele–Shaw cell. Each transparent plate has length L and height H and separation between them D and at time t the rise height is $y(t)$. The cell is under the gravity field $\mathbf{g}$ and has a longitudinal, constant temperature gradient $G = \Delta T/H$. The flat free surface of the liquid is only representative and dA indicates the differential area element.

together, because the characteristic length, D, when the temperature of the fluid will be homogenized, is very small.

It is possible to suppose that during imbibition a Poiseuille-like flow is developed between the parallel plates. So, the capillary pressure is given by $p_c = 2\sigma\cos\theta/D$ where θ is the contact angle. This last approximation is satisfied only when $L \gg D$ [13]. In this case the inner walls shear stress on the wall is given by $\tau_w = 6\mu\bar{u}/D$ where $\bar{u}$ is the average velocity of the flow and μ is the dynamic viscosity [12]. Finally, following the supposition of a constant fluid density ρ it is easy to show that the one-dimensional motion equation is

$$\frac{2\sigma(h(t))}{D}\cos\theta(LD) - \rho g DLh - \int_0^{h(t)} \tau_w\, dA = 0. \tag{1}$$

In Eq. (1) the gravity has been taken into account, and it is considered that the viscous term will be integrated from 0 to $h(t)$ because it acts on each differential element of area $dA = 2L\,dy$, while the surface tension acts only on the whole free surface LD located at $y = h(t)$ (see Fig. 1). Similar procedures have been successfully employed in the studies of isothermal imbibition (see, for instance, Refs. [14,15]).

During imbibition dynamic viscosity varies from point to point along y due to the variations of the temperature on the cell walls, i.e., $\mu = \mu(y)$. Similarly, the surface tension takes a given value depending on the instantaneous height, $h(t)$, of the free surface (imbibition front) where the temperature is $T(y = h) = T_0 + Gh$. These facts and the use of the expression for τ_w given above transforms Eq. (1) into the equation

$$2\sigma\big(h(t)\big)\cos\theta - \rho g Dh - \frac{12}{D}\frac{dh}{dt}\int_0^{h(t)} \mu(y)\, dy = 0. \tag{2}$$

To evaluate the integral term it is convenient, to a first approximation, to employ the Taylor's series for μ and σ given in the form $\mu = \mu_0[1 + 1/\mu_0(d\mu/dT)Gy]$ and $\sigma = \sigma_0[1+1/\sigma_0(d\sigma/dT)Gh]$, where μ_0 and σ_0 are the values at a temperature of reference, $T = T_0$. The substitution of both expressions in Eq. (2) leads to

$$\frac{2D\sigma_0\cos\theta}{12\mu_0} - \frac{[h + \frac{\mu' G h^2}{2\mu_0}]}{[1 + \frac{\sigma' G h}{\sigma_0}]}\frac{dh}{dt} - \frac{\rho g D^2 h}{12\mu_0(1 + \frac{\sigma' G h}{\sigma_0})} = 0, \tag{3}$$

where $\mu' = d\mu/dT$ and $\sigma' = d\sigma/dT$. The previous non-linear differential equation can be expressed in a simpler form, by using the dimensionless variables $\xi = h/h_\infty$ and $\tau = \sigma_0\cos\theta Dt/6\mu_0 h_\infty^2$ where h_∞ is the equilibrium height reached by assuming isothermal imbibition and the balance between the weight and the capillary force, so $h_\infty = 2\sigma_0\cos\theta/(\rho g D)$. Moreover, the following dimensionless parameters are considered

$$A = \frac{\mu' G h_\infty}{2\mu_0}, \qquad B = \frac{\sigma' G h_\infty}{\sigma_0}, \tag{4}$$

the quantities A and B are, respectively, the dimensionless viscosity gradient and the dimensionless surface tension gradient. Therefore, the resulting dimensionless equation is

$$1 + (B-1)\xi - \big(\xi + A\xi^2\big)\frac{d\xi}{d\tau} = 0. \tag{5}$$

Finally, the dimensionless solution under the boundary condition $\xi = 0$ at $\tau = 0$ has the form

$$\tau = \left[\frac{1}{B-1} - \frac{A}{(B-1)^2}\right]\xi + \frac{A}{2(B-1)}\xi^2 + \left[\frac{A}{(B-1)^3} - \frac{1}{(B-1)^2}\right]\ln\big[(B-1)\xi + 1\big]. \tag{6}$$

This last relation gives a direct way to determine the time evolution of the imbibition front by considering explicitly the parameters A and B. Clearly, as will be shown in experiments, these latter quantities are strongly dependent on the selected substances and the magnitude and sign of temperature gradient.

It is easy to found in the isothermal case $G = 0$ ($A = B = 0$) that the solution has the form

$$\tau = -\xi - \ln[1 - \xi], \tag{7}$$

which is the well-known dimensionless form for imbibition in the Hele–Shaw cell under the gravity field [16]. Moreover, the Washburn law is recovered

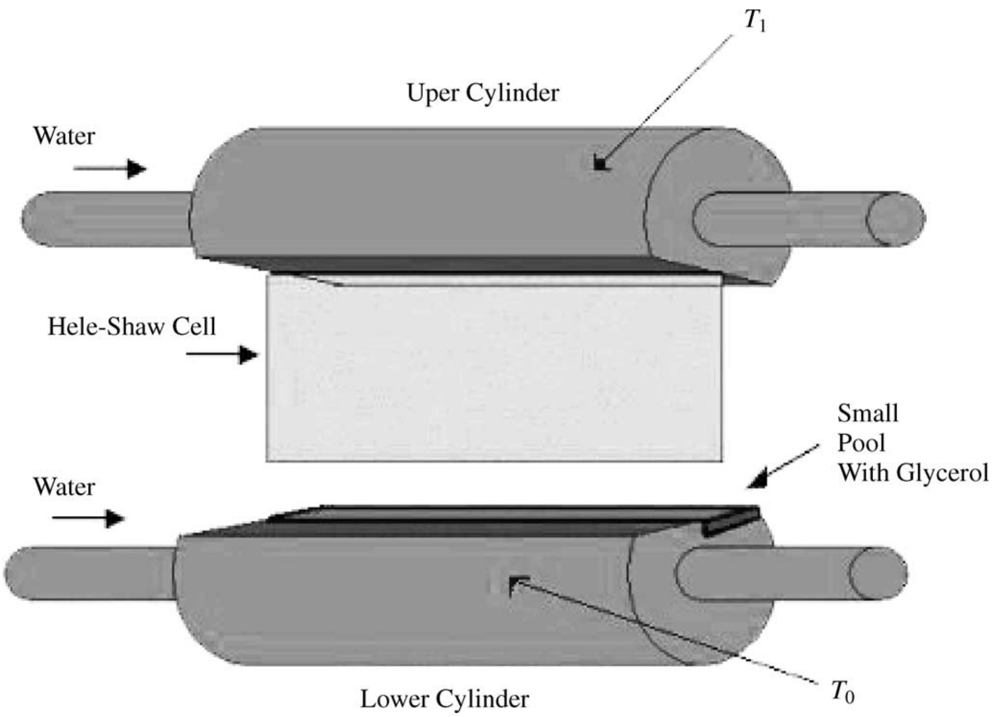

Fig. 2. Schematic view of the experimental setup where the array of the heat exchangers and the Hele–Shaw cell are indicated. Notice that the reservoir filled with glycerol is located at the lower part, alongside the zone of thermal contact region and the cell.

when $\xi \ll 1$ because in this case the asymptotic form of solution (7) is given by $\xi^2 = \tau$ or in dimensional form $h^2 = \sigma_0 D \cos\theta t/(6\mu_0)$ [10].

3. Experiments

Imbibition experiments under temperature gradients were performed in a glass-walled Hele–Shaw cell with length $L = 7.6 \times 10^{-2}$ m, height $H = 2.6 \times 10^{-2}$ m ($L \approx 3H$) and average gap thickness $D = 4.3 \times 10^{-5}$ m. The thickness of each transparent glass plate was 1×10^{-3} m. Experiments were made in two stages. First, temperature gradient was achieved on the glass plates and afterwards the imbibition was induced between the gap by the capillary pressure. The detailed procedure for each stage was carried out as follows: initially, the temperature difference ΔT between the lower (T_0) and upper (T_1) parts of the cell was established when the plates were in contact, for a period of time larger than the diffusive time, with the surfaces of two horizontal copper cylinders which were used as single pass heat exchangers. (See Fig. 2.) A good control of temperatures before and during the imbibition was obtained by employing recirculating thermal baths in a controlled-temperature room at ambient temperature $T_{\mathrm{room}} = 297.2$ K. The room temperature and the temperature distribution on the glass plates were measured by using an infrared thermal camera (model FLIR Thermacam-PM595) whose resolution is ± 0.1 K and its sensitivity is 20 mK at T_{room}.

The glycerol properties (at 299 K) were density $\rho = 1259.9$ kg/m^3 and thermal diffusivity $\alpha_f = 9.35 \times 10^{-8}$ m^2/s and the thermal diffusivity of the glass plates was $\alpha_s = 7.46 \times 10^{-7}$ m^2/s. The contact angle between either glass plate and the liquid was $\theta = 0.57$ rad and was determined by the static sessile drop method [11]. The changes of the surface tension σ and the dynamic viscosity μ with T in glycerol have been plotted in Fig. 3 in the interval $293 \leqslant T \leqslant 323$ K [21]. So, the characteristic time to achieve the gradient along the plates by heat diffusion was $t_{Ds} = L^2/\alpha_s = 15$ min and the diffusive time associated with the homogenization of the temperature in the fluid,

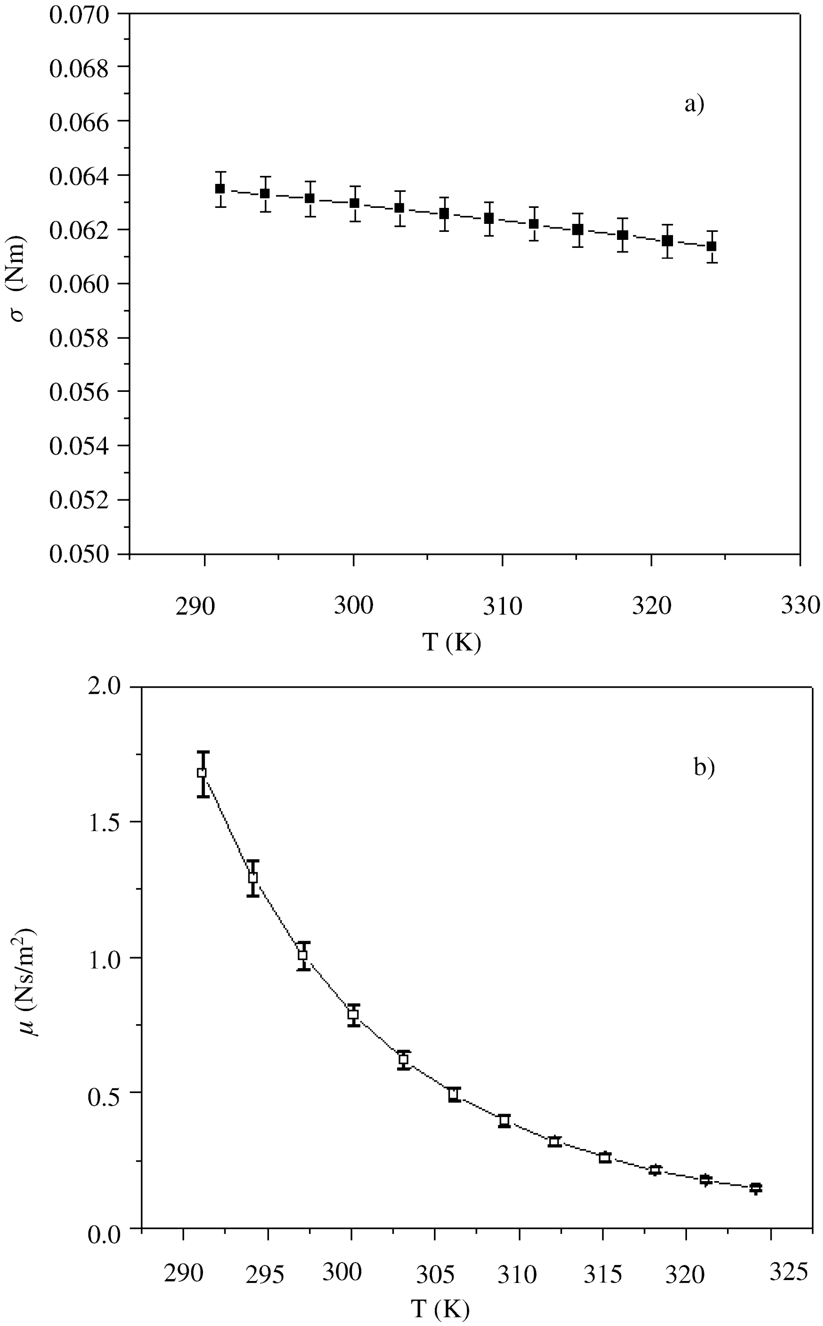

Fig. 3. Plots of (a) surface tension σ and (b) dynamic viscosity μ as a temperature function for glycerol. Data were taken from Ref. [21], error bars reported therein are included.

along D, was $t_{Df} = D^2/\alpha_f$ which was of around $t_{Df} = 0.02$ s. This last period of time is smaller than the shortest imbibition time reached in experiments for a rise height equal to D which has had a value of the order $t_I = 0.2$ s. Meanwhile the steady-state temperature distribution on the plates was reached, at the lower cylinder a reservoir filled with glycerol, alongside the contact zone with the plates, allows the liquid to reach also the temperature $T = T_0$. Therefore, the liquid penetrates into the cell without sudden changes in the temperature because the cell was immediately and gently dipped vertically into this thermalized reservoir and the imbibition front was digitally video recorded.

In experiments, four temperature distributions were analyzed. These were the case of a positive temperature gradient (named G_+) where $G_+ = 300$ K/m, the inverse case of a negative gradient (named G_-) where $G_- = -300$ K/m and two cases with $G = 0$ but where the temperature values were $T_0 = 299$ K and $T_0 = 307$ K. Notice that in order to have direct comparison between cases with negative and positive gradients these were chosen such that $|G_-| = G_+$. The cases where $G = 0$ were obtained when both heat exchangers and the overall Hele–Shaw cell were at the same temperature between them, that is, $T_0 = T_1$. A typical temperature distribution obtained on the glass plates is shown in Fig. 4. This corresponds to the specific case of negative temperature gradient where the temperature was varied from $T_0 = 307$ K up to $T_1 = 299$ K (the temperature values for the case of positive gradient were exactly the opposites). Imbibition began when the cell having a temperature gradient were inserted into the liquid reservoir. The advance of the imbibition front was recorded with a digital video camera and the evolution of the averaged imbibition front, h, was measured each $1/30$ s through the measurement on a PC of the front of advance. Using this method it is possible to measure advance distances as small as 0.5 mm and three independent experiments were made for each value of the gradients above considered. In order to avoid hysteresis phenomena in the realization of each experiment a new couple of clean, glass plates were used.

Fig. 5 depicts the main results for the averaged rise height h as a function of time t obtained for the several values of G (data for $\mu(T)$ and $\sigma(T)$ were taken from Fig. 3). There, a direct comparison between the experimental data (symbols) and theory (curves) is shown. Profiles with $G \neq 0$ were plotted by solving numerically Eq. (6) and those for $G = 0$ were plotted by using Eq. (7). Notice that data fit very well the theoretical curves, however the case for G_+ was underestimated after the middle height of the cell.

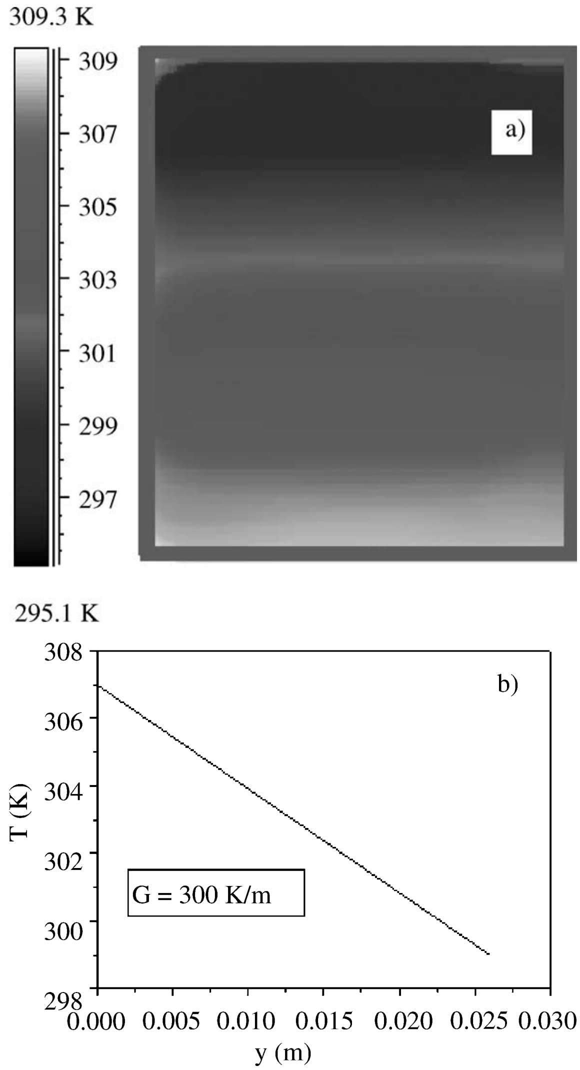

Fig. 4. (a) Thermography and its scale on a glass plate and (b) averaged temperature profile, obtained from (a), on the glass plate as a function of y. Here it is evidenced the existence of a negative, constant temperature gradient. Temperatures were measured in this case at the steady-state regime of heat conduction by imposing $T_0 = 307$ K and $T_1 = 299$ K.

Despite this fact, which is discussed below, the main result was that the evolution of the imbibition fronts in the glycerol-glass system is faster when temperature gradient is positive and are slower in the isotherm and negative gradient cases. The causes of deviation between theory and experiments in the case having positive gradient could be due to many reasons but the most probable are the dependence of contact angle θ

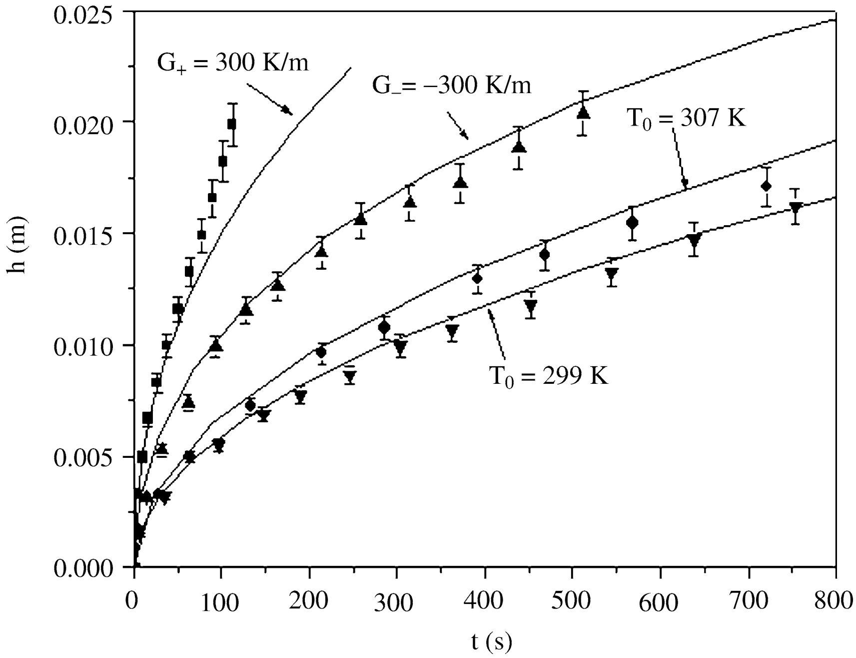

Fig. 5. Dimensional plot of the averaged rise height, h, as a function of time, t, for cases here considered: $G_+ = 300$ K/m (■), $G_- = -300$ K/m (▲), $G = 0$ and $T_0 = T_1 = 299$ K (●) and $G = 0$ and $T_0 = T_1 = 307$ K (▼). Error bars are included.

on velocity [11,15] and on temperature variations [22]. However, further studies in this direction are needed because changes in this quantity should not be ignored to achieve definitive conclusions.

4. Conclusions and remarks

In this work the problem of imbibition under temperature gradients in a capillary Hele–Shaw cell has been analyzed through the use of a motion equation and the comparison with experiments was made. In our case, the study of this problem has been motivated by the ubiquitous presence of imbibition into vertical fractures in subterranean fractured reservoirs. Common sense suggest that the imbibition would be influenced by the geothermal gradients. Thus, the main considerations in the model were the imposition of an overall constant temperature gradient on the plates and those related to the dependence of the dynamic viscosity and surface tension on temperature. As a result, it was found that the imposed temperature gradient has a crucial influence on the evolution of the average imbibition front which depends importantly on the sign and the way in which the dimensionless viscosity gradient A evolves respect to the dimensionless surface tension gradient B. Another important issue is that the asymptotic limit $\xi \to 0$, which is associated with the limit of short rise distances (Washburn law), is fully recovered when $G = 0$. Under adequate changes, present study also may be useful to understand quantitatively the origin of the increase in the oil recovery rate at high temperatures which has been evidenced in experiments using hot porous samples and liquids [19,20]. Such a increase is, in this stage, evidently ascribed to the strong decrease of viscosity in comparison with the slow changes in the surface tension when temperature increases.

Finally, under the approximation of one-dimensional models, the possible existence and nature of interfacial instabilities in imbibition under temperature gradient cannot further studied, however, the understanding of these instabilities can be of importance to achieve a better knowledge of the physics involved in these processes [16,23]. Work along this line is under progress.

Acknowledgements

This work has been made under the support of projects FIES 98-58-I and D.2100 (IMP). A.M. acknowledges Spanish Ministerio de Educación Cultura y Deporte Project SAB2002-0137.

References

[1] V.G. Levich, Physicochemical Hydrodynamics, Prentice-Hall, Englewood Cliffs, NJ, 1962.
[2] A.A. Nepomnyashchy, M. Velarde, P. Collinet, Interfacial Phenomena and Convection, Chapman and Hall/CRC, London, 2002.
[3] V. Ludviksson, E.N. Lightfoot, AIChE J. 17 (1971) 1166.
[4] G.L. Batten, J. Colloid Interface Sci. 102 (1984) 513.
[5] T.S. Sammarco, M.A. Burns, AIChE J. 45 (1999) 350.
[6] D.E. Kataoka, S.M. Troian, Nature 402 (1999) 794.
[7] A. Munch, SIAM J. Appl. Math. 62 (2002) 2045.
[8] S.D. Paterna, M. Lappa, J. Fluid Mech. 479 (2003) 307.
[9] Ch. Nienhuser, H.C. Kuhlman, J. Fluid Mech. 458 (2002) 35.
[10] E.W. Washburn, Phys. Rev. 17 (1921) 273.
[11] S. Middleman, Modeling Axisymmetric Flows, Academic Press, New York, 1995.
[12] F.M. White, Fluid Mechanics, McGraw–Hill, New York, 1994.
[13] C. Isenberg, The Science of Soap Films and Soap Bubbles, Dover, New York, 1992.
[14] J.M. Bell, F.K. Cameron, J. Phys. Chem. 10 (1906) 658.
[15] A. Hamraoui, K. Thuresson, T. Nylander, V. Yaminsky, J. Colloid Interface Sci. 226 (2000) 199.
[16] M. Dubé, S. Majaniemi, M. Rost, M.J. Alava, K.R. Elder, T. Ala-Nissila, Phys. Rev. E 64 (2001) 051605.
[17] N.R. Morrow, G. Mason, Curr. Opin. Colloid Interface Sci. 6 (2001) 321.
[18] O.M. Phillips, Flow and Reactions in Permeable Rocks, Cambridge Univ. Press, Cambridge, 1991.
[19] T. Babadagli, J. Pet. Sci. Eng. 14 (1996) 197.
[20] T. Babadagli, J. Pet. Sci. Eng. 33 (2002) 223.
[21] F.P. Incropera, D.P. Dewitt, Fundamentals of Heat and Mass Transfer, Wiley, New York, 2001.
[22] H.Y. She, B.E. Sleep, Water Resour. Res. 34 (1998) 2587.
[23] A. Hernández-Machado, J. Soriano, A.M. Lacasta, M.A. Rodríguez, L. Ramírez-Piscina, J. Ortín, Europhys. Lett. 55 (2001) 194.