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Equilibrium height of a liquid in a gap between corrugated walls under spontaneous capillary penetration

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A simple and general method is presented to calculate the equilibrium surface of a liquid that penetrates spontaneously, due to capillarity, in the gap between two vertical corrugated plates. Several properties of the equilibrium solution are discussed and the results are backed by a qualitative experiment.

1. Introduction

This work considers the problem of the equilibrium height reached by a liquid during its spontaneous capillary penetration into a gap between two parallel vertical corrugated plates. This configuration can be considered as a simple idealization of a vertical fracture in a real rock of an oil reservoir saturated with two immiscible fluids, for which the irregular separation between the inner surfaces of the fractured rock creates channels where capillary penetration occurs and leads to complex equilibrium free surfaces [1,2]. Similarly, but in another context, micro-moulding in capillaries (MIMIC) [3] uses capillary penetration of polymeric liquids [4] or melt ceramics [5] in complex micro- and nano-channels, and this type of study can be useful to design and build solid micro- and nano-structures.

The study of the equilibrium height of a liquid rising in a gap was initiated nearly three centuries ago by the pioneering works of Taylor [6] and Hauksbee [7], with their experiments on the hyperbolic shape of the equilibrium meniscus in the gap between two vertical plates that form a small angle. The evolution of the liquid between the plates toward its equilibrium has been recently analyzed in the case when viscous forces dominate the motion of the liquid [8]. The equilibrium height in the gaps between vertical cylindrical fibers has also been carefully analyzed under several configurations [9–11]. Another interesting case occurs in a capillary of square cross-section, in which the elevation of the liquid is greater near the corners than at the center of the capillary, and an approximate analytical expression for the equilibrium shape has been found [10]. In terms of the mass of liquid, the error introduced in the calculations when the capillary elevation in the corners is left aside is around 6%. As another example of the variety of equilibrium heights reached in capillaries, it is appropriate to mention that in convergent conical capillaries two equilibrium heights are possible but only one equilibrium height occurs in divergent cones [12,13].

The problem of the equilibrium height of a liquid between corrugated plates has been previously analyzed using variational methods that minimize the local potential energy [14,15]. Unfortunately, the resulting solution is erroneous since it predicts that the equilibrium height reaches a maximum where the separation of the plates is largest. This is not physically possible because the capillary pressure is inversely proportional to the local separation of the plates.

The objective of this work is to correctly compute the equilibrium height in the gap between two vertical corrugated plates. A set of experiments has been conducted to validate the theoretical results for both horizontal and tilted corrugations. The structure of the paper is as follows: in the following section the geometry of the corrugated cell is defined and the problem of the equilibrium height is formulated. An analytical expression for the equilibrium height is derived in Section 3 from a balance of pressures at the liquid surface. Experiments carried out with corrugated surfaces in order to show the equilibrium profiles and some of their main characteristics are discussed in Section 4. Finally, Section 5 summarizes the main conclusions of the work.

2. The problem

Consider first the simple calculation of the equilibrium height of a meniscus in the open gap between two parallel vertical flat

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plates separated a distance $2w$. The liquid has density ρ and surface tension σ, and it wets the plates with a contact angle $\theta < \pi/2$. The capillary pressure causing the liquid to ascend between the plates is $p_c = \sigma \cos\theta / w$, and the hydrostatic depression that is generated under the gravitational field when the height of the liquid is H_{fp} is $p_g = \rho g H_{fp}$, where g is the acceleration due to gravity. The equilibrium height obtained by equating both pressures is

$$H_{fp} = \frac{\sigma \cos\theta}{\rho g w}, \tag{1}$$

which is known as Jurin's law [16].

Consider now the gap between two vertical corrugated plates. Let x and y denote horizontal and vertical distances in the center plane of the gap, with $y = 0$ at the level of the liquid outside the gap. Assume that the two vertical plates are slightly corrugated in a direction making an angle α to the horizontal, so that the width of the gap is $2h(x, y)$ with

$$h(x,y) = w\left[1 - (1-\delta)\cos\frac{2\pi}{\lambda}(x\cos\alpha + y\sin\alpha)\right]. \tag{2}$$

Here $w(1-\delta)$ and λ are the amplitude and wavelength of the corrugation. Flat parallel plates a distance $2w$ apart are obtained for $\delta = 1$, and the amplitude of the corrugation is maximum for $\delta = 0$.

Fig. 1A is a sketch of the periodically corrugated plates and the liquid between them in the case $\alpha = 0$ of a horizontal corrugation. Fig. 1B shows the equilibrium profile of the liquid. Here $H(x)$ is the intersection of the liquid surface with the center plane of the gap. Fig. 1C is a horizontal section of the gap showing the horizontal corrugation of the plates.

The mean separation between the plates, $2w$, is assumed to be small compared to the capillary length $l_c = \sqrt{\sigma/\rho g}$ and the wavelength λ; *i.e.*, $2w \ll (l_c, \lambda)$. In these conditions, the curvature of the liquid surface generated by the corrugation is of order $w/\lambda^2 \ll 1/w$, and the normal section of this surface by a plane perpendicular to the center plane of the gap is nearly an arc of a circle of radius $R = h/\cos\theta$.

3. Equilibrium heights

The pressure of the liquid immediately underneath its surface, at $y = H(x)$, is

$$p_a - \frac{\sigma\cos\theta}{h(x,H)},$$

where p_a is the pressure of the gas surrounding the plates and $h(x, y)$ is given by (2). The hydrostatic pressure of the liquid at the equilibrium height $H(x)$ is

$$p_a - \rho g H.$$

Equating these two pressures we find the equation

$$H\left[1 - (1-\delta)\cos\frac{2\pi}{\lambda}(x\cos\alpha + H\sin\alpha)\right] = \frac{\sigma\cos\theta}{\rho g w}, \tag{3}$$

for the equilibrium height.

This equation can be explicitly solved in the case $\alpha = 0$ of a horizontal corrugation, for which

$$H(x) = \frac{H_{fp}}{1 - (1-\delta)\cos 2\pi x/\lambda}, \tag{4}$$

where H_{fp} is the uniform height of the liquid between flat plates given by (1). The quantity H/H_{fp} is shown in Fig. 2 as a function of x/λ for $\delta = 0.1$, 0.4 and 1. The maximum and minimum heights of the liquid and the points where they are attained are

$$H_{\max} = \frac{H_{fp}}{\delta}, \quad \text{at } x = \pm n\lambda, \tag{5}$$
$$H_{\min} = \frac{H_{fp}}{2-\delta}, \quad \text{at } x = \pm\left(\frac{1}{2} + n\right)\lambda,$$

with $n = 0, 1, 2, \ldots$

The volume of liquid contained in one wavelength of the corrugation is

$$V_0 = \int_0^\lambda 2h(x)H(x)dx = \frac{2\lambda\sigma\cos\theta}{\rho g}, \tag{6}$$

which is independent of the amplitude of the corrugation. The increase of volume (compared to that for vertical plates) in the re-

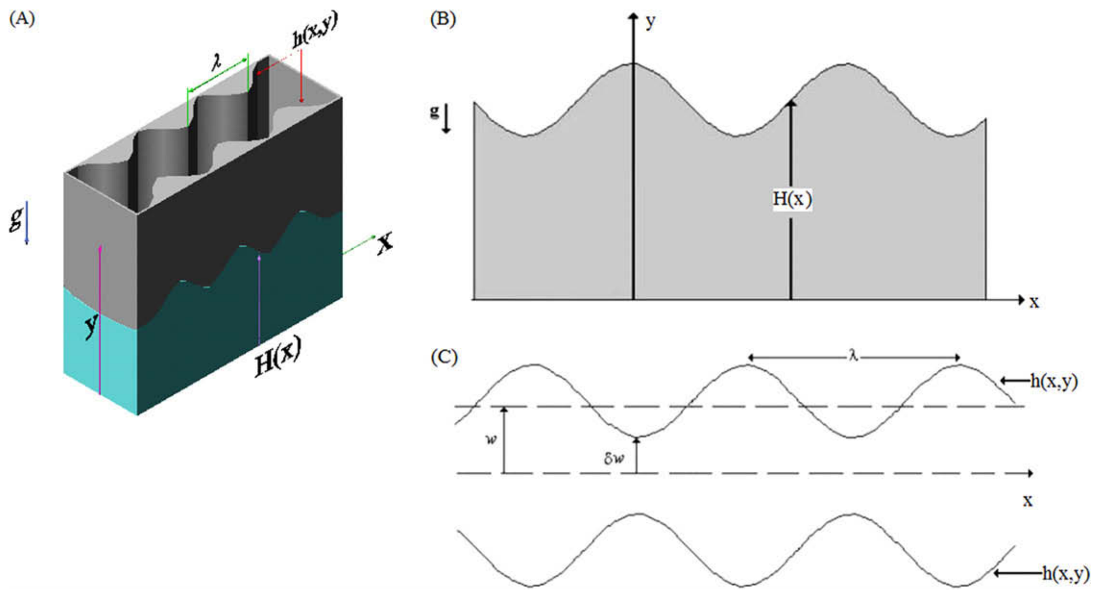

Fig. 1. (A) Sketch of the gap between two corrugated plates where a liquid penetrates spontaneously by capillarity. (B) Sketch of the equilibrium position of the liquid. (C) Cross-section of the corrugated plates.

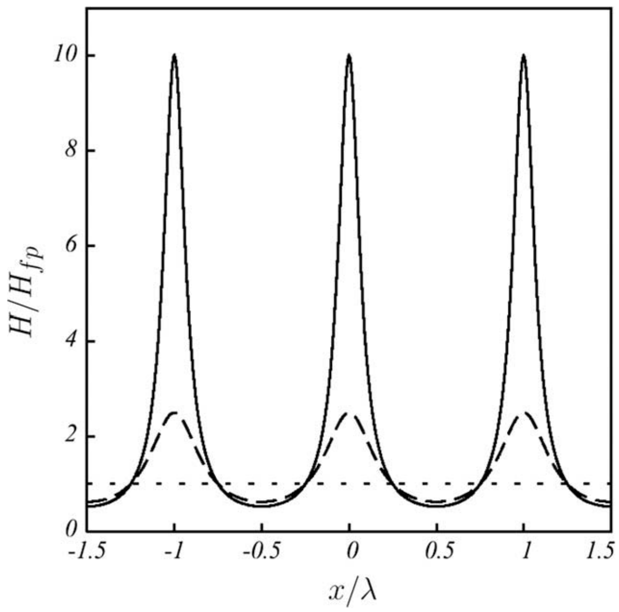

Fig. 2. Equilibrium profiles for a horizontal corrugation with $\delta = 0.1$ (solid), 0.4 (dashed) and 1 (dotted).

gions of large elevation where the gap is narrow is exactly balanced by the decrease of volume in the regions of small elevation where the gap is wide. This result was kindly pointed out to the authors by an anonymous referee.

We turn now to the general case of an inclined corrugation. Eq. (3) defines H/H_{fp} as a function of $(x/\lambda)\cos\alpha$ and the two dimensionless parameters δ and $(H_{fp}/\lambda)\sin\alpha$. Fig. 3 illustrates this result for $(H_{fp}/\lambda)\sin\alpha = 0.15$ and three values of δ. The distribution of height is more complex than for a horizontal corrugation. However, the height of the liquid on a line of constant gap width is the same in both cases. The maximum and minimum heights are therefore the same as when $\alpha = 0$ and are attained at the locations of minimum and maximum gap width. The volume of liquid in one wavelength of the corrugation is

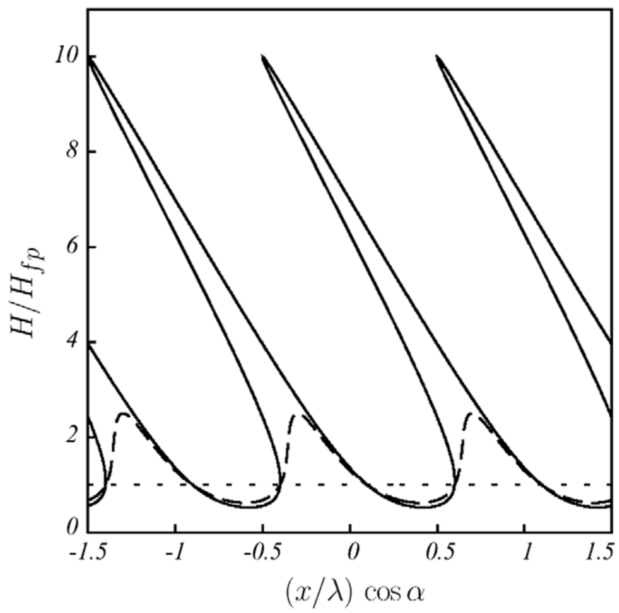

Fig. 3. Equilibrium profiles for an inclined corrugation with $(H_{fp}/\lambda)\sin\alpha = 0.15$ and $\delta = 0.1$ (solid), 0.4 (dashed) and 1 (dotted).

$$V = \frac{V_0}{\cos\alpha}. \tag{7}$$

This volume is larger than the volume for a horizontal corrugation of the same wavelength [V_0 in (6)]. The effect is the same as when the whole channel with a horizontal corrugation is tilted an angle α about the x axis. As in that case, the volume (7) diverges when $\alpha \to \pi/2$ for simple geometrical reasons; namely, the distance traversed by the meniscus along a line of constant gap width before it reaches its equilibrium position increases as $1/(\pi/2 - \alpha)$.

The results of this section are largely independent of the shape of the periodic corrugation. A sinusoidal shape has been assumed in (2) mostly for definiteness.

4. Experiments

The complexity and richness of the equilibrium profiles predicted can be illustrated by a set of qualitative experiments. Corrugated cells were made out of commercial corrugated plastic sheets 60 mm in length and 70 mm in height, with corrugations parallel to the vertical edges or making an angle of 46° with them. The corrugations are not sinusoidal; they are cylindrical protuberances of nearly semi-circular cross-section in the plane of the sheet (see Fig. 4A). The corrugations of two parallel plastic sheets were aligned to approach the configuration sketched in Fig. 1. The wavelength of the corrugations is $\lambda = 2.13 \times 10^{-3}$ m and the height of the protuberances is $h_m = 30$ μm. The mean width of the gap between the plastic sheets is $2w = 300$ μm, which nearly coincides with the distance between the sheets outside of the protuberances, given the small value of h_m/λ. In these conditions, $\delta = (w - h_m)/w = 0.8$. The liquid used in the experiments was a silicone oil (from Brookfield) with $\mu = 500$ cP, $\rho = 970$ kg/ m^3, and $\sigma = 0.021$ N/m, which wets the plastic sheets with a contact angle $\theta = 36°$.

Fig. 4B and C show the equilibrium shape of the liquid surface between corrugated sheets for $\alpha = 0°$ (horizontal corrugation) and 46°, respectively. Only a region around the surface of the liquid is shown. The level of the liquid in the reservoir where the lower edges of the sheets are immersed cannot be clearly seen in the photos due to reflections in the meniscus at the outer sides of the sheets, and therefore we have not included it in the figures. The minimum height of the liquid, $H_{\min}$, was independently measured in the experiments and found to be in good agreement with the theoretical value $H_{\min} \approx \sigma\cos\theta/\rho g w = 1.19 \times 10^{-2}$ m. However, only the difference of heights $\Delta H = H_{\max} - H_{\min}$, which can be seen in Fig. 4B and C, is discussed in what follows.

The maximum height is attained at the locations of minimum half-separation $w - h_m = 120$ μm in all the cases. Four maxima and minima are visible in Fig. 4B. The average of the measurements of ΔH for the three central couples of maxima and minima in the figure is $\overline{\Delta H} = 3.1 \times 10^{-3}$ m, which is only about 7% higher than the theoretical value $\Delta H = 2.9 \times 10^{-3}$ m. The shape of the profile $H(x)$ is qualitatively similar to that in Fig. 2, but not identical to it due to the large difference between $h(x)$ and the sinusoidal function in (2). The volume of liquid in the cell per wavelength of the corrugation (not fully visible in Fig. 4B) is also in good agreement with V_0 in (6).

Experimental results for a corrugation at an angle $\alpha \simeq 46°$ to the horizontal with the same values of λ and δ as before are shown in Fig. 4C. Two maxima and minima are now visible, and the average height difference is $\overline{\Delta H} = 3.1 \times 10^{-3}$ m, which again is approximately 7% above the theoretical value $\Delta H = 2.9 \times 10^{-3}$ m. Neither the equilibrium heights nor ΔH are substantially affected by the inclination of the corrugation, and the volume of liquid in the cell increases in qualitative agreement with (7).

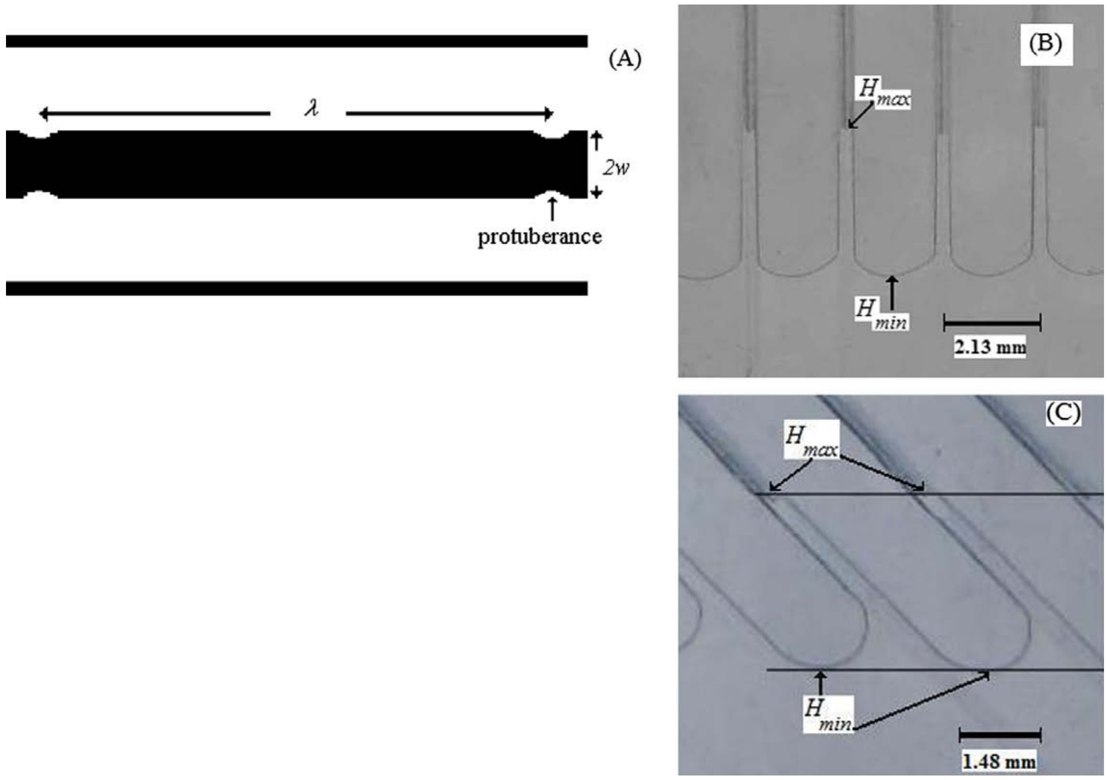

Fig. 4. (A) Picture of a section of the gap (black central region) between a couple of corrugated sheets. Protuberances have a height $h_m = 30$ μm and are spaced a distance $\lambda = 2.13$ mm. (B) Equilibrium profile of the liquid in a gap with vertical protuberances. (C) Equilibrium profile of the liquid in a gap with tilted protuberances. The locations of H_{max} and H_{min} are indicated in each panel, together with length scales.

5. Conclusions

The equilibrium profile reached by spontaneous penetration of a liquid in the gap between two corrugated plates has been computed. Theoretical results show that, in the presence of corrugation, capillary penetration may yield complex equilibrium profiles with very different maximum and minimum heights. Experiments using a periodically corrugated cell show that the model here presented provides a good description of the equilibrium profiles. The results reported are of interest to understand some aspects of liquid trapping in rock fractures and also can be of utility to mould, by capillarity, micro- and nano-structures by employing the cure of photo sensible polymers.

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