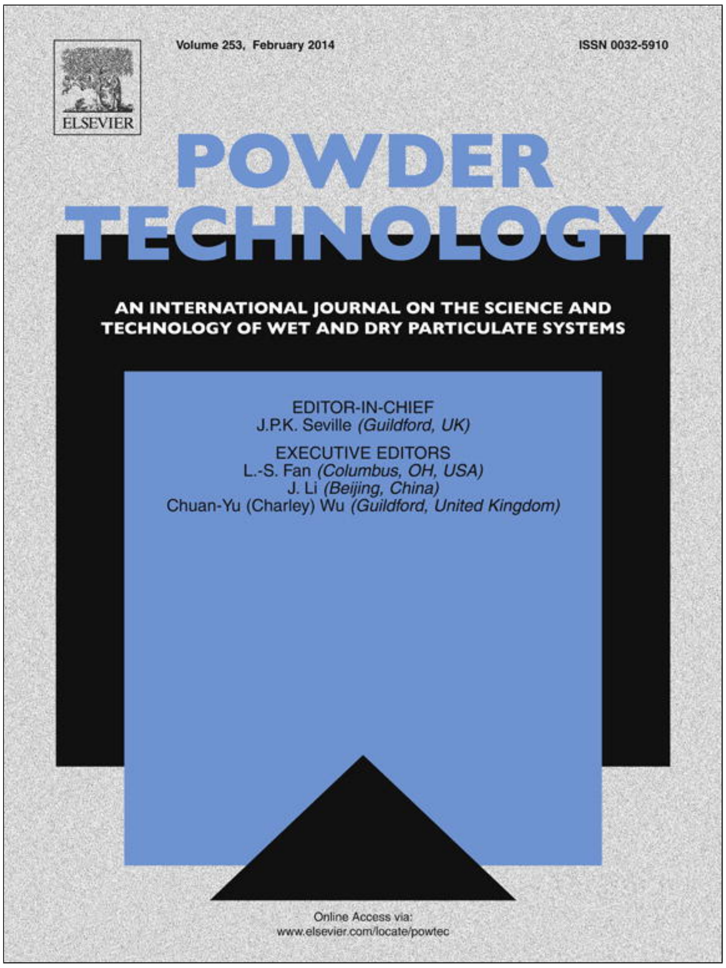
Volume 253, February 2014
ISSN 0032-5910
ELSEVIER
POWDER TECHNOLOGY
AN INTERNATIONAL JOURNAL ON THE SCIENCE AND TECHNOLOGY OF WET AND DRY PARTICULATE SYSTEMS
EDITOR-IN-CHIEF
J.P.K. Seville (Guildford, UK)
EXECUTIVE EDITORS
L.-S. Fan (Columbus, OH, USA)
J. Li (Beijing, China)
Chuan-Yu (Charley) Wu (Guildford, United Kingdom)
Online Access via:
www.elsevier.com/locate/powtec

Powder Technology 253 (2014) 270–275

Contents lists available at ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Discharge rates of dry granular material from bins with lateral exit holes

A. Medina [a], D. Cabrera [a], A. López-Villa [a], M. Pliego [b]

[a] SEPI ESIME Azcapotzalco, Instituto Politécnico Nacional, Av. de las Granjas 682, Col. Santa Catarina, 02250 Azcapotzalco, D.F., Mexico
[b] DCB, Instituto Tecnológico de Querétaro, Av. Tecnológico s/n esq. M. Escobedo, Col. Centro, 76000, Querétaro, Qro., Mexico

ARTICLE INFO

Article history:
Received 9 May 2013
Received in revised form 2 November 2013
Accepted 17 November 2013
Available online 28 November 2013

Keywords:
Discharge rates
Bins
Angle of repose
Hagen law
Lateral outflow

ABSTRACT

In this work we have analyzed experimentally the mass flow rate $\dot{m}$ of the lateral outflow of cohesionless sand beach and granulated sugar through circular orifices of diameter D and rectangular and triangular slots of hydraulic diameter D_H made in vertical walls of bins. Such experiments were also performed in order to determine the influence of the wall thickness of the bin, w. Geometrical and physical arguments are given to get a general correlation for $\dot{m}$ embracing both quantities, D (or D_H) and w. The angle of repose is also an important factor characterizing these gravity flows.

1. Introduction

The discharge of non cohesive granular materials, from the bottom exits of silos and bins, due to gravity action alone, is a very interesting phenomenon that involves dynamical events as jamming [1], density fluctuations [2] and industrial controlled flows such as dosage of powders and granules [3], among others.

The fundamental study of the law that obeys the mass flow rate of grains, $\dot{m}$ owed to Hagen [4] who, by 1852, performed experiments with sand and he found that $\dot{m}\sim\rho g^{1/2}D^{5/2}$, where ρ is the bulk density, g is the acceleration due to gravity, and D is the diameter of the circular orifice. Thus, the level of filling is not important. This result contrasts with the one occurring in liquids where the mass flow rate depends essentially on the level of filling above the discharge orifice. Actually, in specialized literature other correlations for discharge rates from silos with small openings [5] and hoppers filled with fine and coarse non-cohesive granular materials [6] have been reported.

Moreover, in their seminal paper on the flow of granular solids through circular orifices and slots Beverloo, Leniger and van de Velde [7] have reported other correlations for $\dot{m}$. Despite that, they found that the most suitable correlation to predict the mass flow rates from bottom exits in open-top bins, silos and hoppers is the so called Hagen–Beverloo correlation [7], which has the form $\dot{m}\sim\rho g^{1/2}(D-kd_g)^{5/2}$ where d_g is the mean diameter of the grain and k is a dimensionless constant with typical values k ~ 1–2 [8]. Hence, if $D/d_g \gg 1$ the Hagen formula predicts the discharge rates pretty well.

A very important observation is that several authors studied the discharge rates through slots with triangular and rectangular cross-sections [7,9]. In such case the correlation that best fits the experimental data has the form $\dot{m}\sim\rho g^{1/2}(D_H-kd_g)^{5/2}$ where D_H is the hydraulic diameter (D_H = four times the area of the aperture divided by the perimeter) [7]; the concept of hydraulic diameter has been inherited from fluid mechanics [10].

Despite the enormous utility of Hagen's law (or its generalization, the Hagen–Beverloo law), only a few studies have been conducted to test its validity in the discharge of grains through orifices in the vertical walls of bins [11–17]. The aim of this work is to study experimentally the discharge rates of granular solids through vertical walls of open-top bins for different orifice sizes, D, different cross-sections of orifices and several wall thicknesses, w. As it will be seen later, the thickness of the wall can be used to control the discharge rate and even the granular flow can be arrested if w exceeds a critical value that depends also on the angle of repose of the granular material. Consequently, the term *wall thickness* will be used here assuming that the condition $w/d_g \gg 1$ is fulfilled, *i.e.*, when the average grain size is very small in comparison with w and, at the same time, the angle of repose (the steepest angle of descent of the slope relative to the horizontal plane when material on the slope face is on the verge of sliding) is well defined.

The plan of this work is as follows. Firstly, in the next section we review experimental studies of the mass flow rate from orifices in lateral walls of bins. Then, in Section 3, we report experiments of discharge rates from bottom and lateral exit holes, where the influence of D, D_H and w is examined. Then we propose, on the basis of our experimental results, a correlation that embraces both changes in D (or D_H)

E-mail addresses: amedinao@ipn.mx (A. Medina), main-men@hotmail.com (D. Cabrera), ablopezv@ipn.mx (A. López-Villa), mpliego@mail.itq.edu.mx (M. Pliego).

0032-5910/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.powtec.2013.11.027

and w, in the case of lateral holes, and finally, in Section 4, we give the main conclusions of the study here tackled.

2. Background

To the best of our knowledge, Bagrintsev and Koshkovskii [11] were the first researchers who studied experimentally the problem of the gravity driven lateral outflow of granular material in cylindrical bins with vertical walls. They used oval and circular exit holes made in transparent plastic walls, and observed that "the outflow capacity decreases as wall thickness increases". Later on, experiments in silos with rectangular exit holes [12–14] and circular exit holes [13,15,16] were reported. In summary, these and other authors found that $\dot{m}\sim\rho g^{1/2}D^{n}$, with $n = 3.4$ [11], 3.3 [15,16], $2.6 < n < 3$ [13] and 2.5 [12,17]. It is important to note that in some studies [11–13], D is essentially the hydraulic diameter. Nevertheless, none of the referred works have analyzed systematically the effect of the wall thickness on $\dot{m}$.

3. Experiments

3.1. Bottom holes

In the present work, the simultaneous effects of the wall thickness w and the orifice diameter D on the mass flow rate $\dot{m}$ are examined. At a first glance it is easy to conclude that if the vertical wall is very thick, there will be no efflux of granular material.

On the other hand it is well known that for granular solids the wall thickness does not affect substantially the value of $\dot{m}_0$, the mass flow rate when the exit hole is located at the bottom of a silo (the opposite is common for porous aeratable powders, where aeration develops flow-obstructions near the exit holes [18,19]). In a first instance, we did experiments to measure $\dot{m}_0$ with sand beach (composed of irregular grains of mean diameter $d_g = 0.03$ cm, bulk density $\rho = 1.5 \pm 0.01$ g/cm^3 and angle of repose $\theta_r = 33° \pm 0.5° = 0.57 \pm 0.008$ rad; incidentally, Hagen [4] reported the same value for the bulk density) and granulated sugar (composed of grains of crystal-like shapes of mean diameter $d_g = 0.073$ cm, bulk density $\rho = 0.84 \pm 0.01$ g/cm^3 and angle of repose $\theta_r = 33.5° \pm 0.5° = 0.58 \pm 0.008$ rad; more mechanical properties could be found, for instance, in [20]) that crosses through circular orifices of diameters $D = 0.6$, 0.7, 0.8 and 1 cm and rectangular and triangular slots in an acrylic-made box with a thin wall and a thick wall. Dry sand and granulated sugar are familiar examples of free-flowing bulk solids and this was the main reason to use these materials in our work. The section of the laboratory in which the experiments were done was climate controlled (25 ± 1 °C and 45 ± 10% R.H.). The moisture contents of sand and granulated sugar samples were 0.50 ± 0.06% and 0.015 ± 0.005% w.b., respectively.

In Fig. 1 we show pictures of the samples of sand (Fig. 1(a)) and granulated sugar (Fig. 1(b)) used in our experiments. Pictures were obtained using a Steindorff digital microscope and the corresponding particle size distributions (Fig. 2) were determined using the microscope software which allowed us to find the surface area of each particle and its surface diameter. This method yields the average (median) particle sizes which were 0.03 cm for sand and 0.073 cm for sugar.

Experiments were made upon a transparent box, 10 × 10 cm^2 inner cross-section and 50 cm height. In experiments with bottom exits we have used sand and granulated sugar in bins with several bottom wall thicknesses $w = 0.3$ cm and $w = 0.9$ cm. Details of the measurement procedure of the discharge rates are given afterwards. In Fig. 3 we show the experimental plots of $\dot{m}_0$, as a function of $\rho g^{1/2}D^{5/2}$, for both materials. In this figure we observe that both cases fit very well the Hagen's law (straight lines), and thus the effect of w is not observed. Hence, the relation that fits the experimental data has the form

$$\dot{m}_0 = a\rho g^{1/2}D^{5/2}, \quad (1)$$

where the dimensionless discharge coefficient has the value $a = 0.48$ for sand and $a = 0.46$ for sugar. Here it is important to comment that it has been reported that a may be dependent on the coefficient of friction of the granular material, μ [5,21], on the geometrical characteristics of the silo [5,21], on the grain shapes [22] and on the orifice shape [13]. So far, a may be different for different materials.

In a second series of experiments we have made measurements of the discharge rates of sand and sugar across four horizontal slots with cross-sections having equilateral triangular shapes with lengths $l = 1$, 1.5, 2 and 2.5 cm, respectively. The hydraulic diameters of these slots were computed by using the relation $D_H = l/\sqrt{3}$ [7,10]. Similarly, four slots with rectangular shapes were made, $e = 1$ cm width and $f = 1.5$, 2, 2.5 and 3 cm heights, in these cases $D_H = 2(ef)/(e + f)$. All these slots were made on the bin walls with a thickness of 1.2 and 1.8 cm, respectively. In Fig. 4 we show plots of the mass flow rates for both types of slots, as a function of $\rho g^{1/2}D_H^{5/2}$. It is easily appreciated that the best fit obeys the relation

$$\dot{m}_0 = a_H\rho g^{1/2}D_H^{5/2}, \quad (2)$$

where for bins filled with sand and slots with rectangular cross-sections the slope yields $a_H = 0.89$ and for triangular cross-sections the slope is

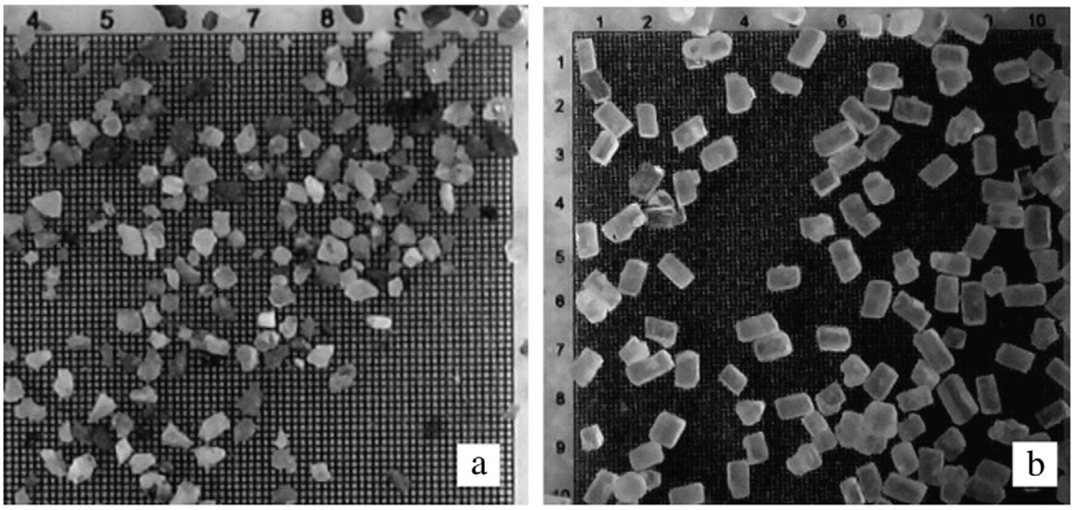

Fig. 1. Micrographs of the particle shapes of samples of sand (a) and granulated sugar (b) used in our experiments. Distance among numbers is 1 mm.

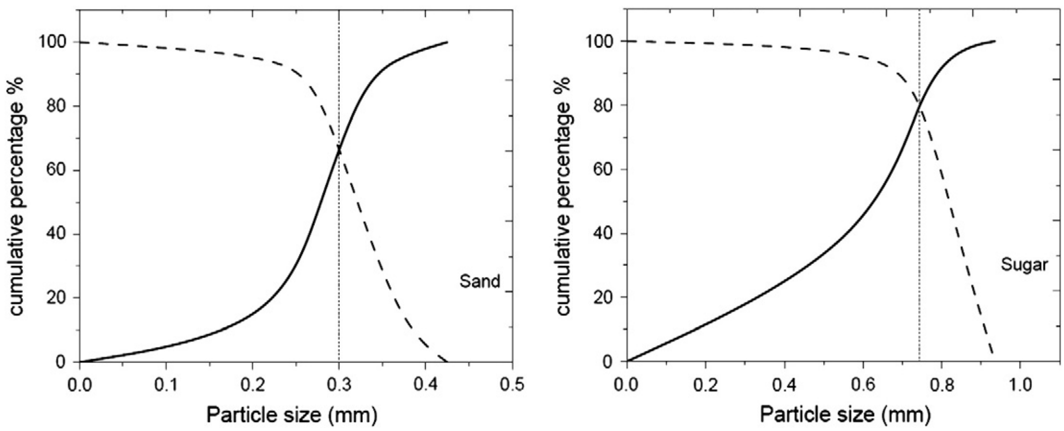

Fig. 2. Plot of the cumulative percentage under- and over-size. The average diameter is given by the median (vertical straight line). The average diameter of the grains of sand is 0.03 cm and of granulated sugar is 0.073 cm.

$a_H = 0.98$. Similarly, for bins filled with granulated sugar $a_H = 0.84$, for rectangles, and $a_H = 0.96$ for triangles. Differences among the values of the discharge coefficients for different shapes of orifices were reported also elsewhere [13].

Thus, in both cases the Hagen law is valid and the correlations (1) and (2) will be very useful to reach a general law of the discharge rates across lateral exit holes.

3.2. Lateral holes

3.2.1. Circular orifices

In Fig. 5(a) the positions of the staggered circular orifices of different diameter D are sketched. There, orifices were made at the middle of each wall. Diameters of the exit holes were: $D = 0.6$ cm at a height $H = 5$ cm from the bottom, $D = 0.7$ cm at $H = 15$ cm, $D = 0.8$ cm at $H = 25$ cm, and $D = 1.0$ at $H = 35$ cm. The circular orifices were made on each wall of the bin; in these experiments four different wall thicknesses were used: $w = 0.3$, 0.4, 0.6 and 0.9 cm. In Fig. 5(b) we show a top view of the bin with the four different wall thicknesses. Thus, an exit hole of a given diameter is at the same height H on each wall, but each wall has a different thickness.

The experimental procedure employed to get the mass flow rates was the following: a reservoir attached to a force sensor model Pasco CI-6537 with a resolution of 0.03 N was located close to the hole and data of weights produced during the discharge of grains were acquired each 0.1 s for a given hole on each wall, *i.e.*, in each experimental run only one orifice was opened.

To understand our measurement procedure it is crucial to quantify the involved forces in the fall of grains. In quantitative terms, suppose that the sand (or sugar) is dropping at a steady rate of $\dot{m}$ g/s and that it takes t_1 s for the particles to hit the reservoir. The velocity of grains hitting the compartment would be essentially $v = gt$ (in the simplest case we have neglected the initial velocity of grains just at the exit of the hole). In an infinitesimal time, dt, the change in momentum experienced by the reservoir would be $v\dot{m}dt$, so that the impulsive force (weight, W) experienced by the reservoir would be $W = v\dot{m}dt/dt = \dot{m}v = \dot{m}gt$. This force corresponds to a linear increase in the weight in the reservoir during a time of measurement t_m, longer than t_1. Finally, we can get $\dot{m} = (dW/dt)/g$. Actually, $\dot{m}$ is near a constant and the time-average of $\dot{m}$ is the quantity that we report as the mass flow rate. For example, the mean value of this quantity was computed from ten independent measurements for each couple of values (D,w). It is important to comment that no dependence on the level of filling of the silos was detected in our mass flow rate measurements.

As it was mentioned in the background, the wall thickness affects the outflow, *i.e.*, w also modulates $\dot{m}$. In Fig. 6 we give snapshots showing the outflows through a thin (left-hand side) and a thick (right-hand

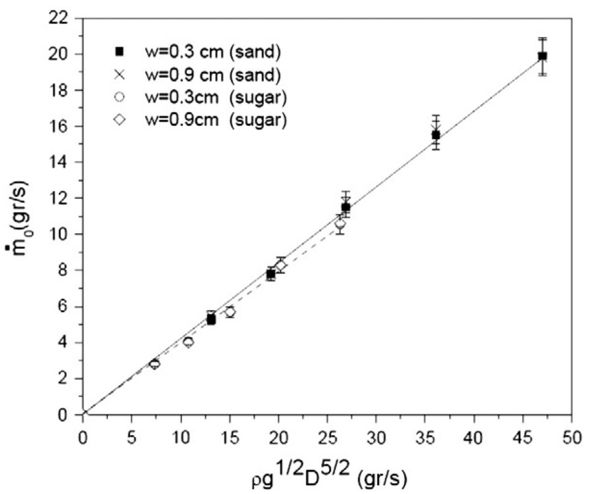

Fig. 3. Mass flow rate through horizontal circular orifices drilled in acrylic plates of thickness $w = 0.3$ cm and $w = 0.9$ cm, $\dot{m}_0$, as a function of $\rho g^{1/2}D^{5/2}$. Data fit the Hagen's law, Eq. (1), with $a = 0.48$ for sand (solid line) and $a = 0.46$ for granulated sugar (dashed line). Error bars are of 4%.

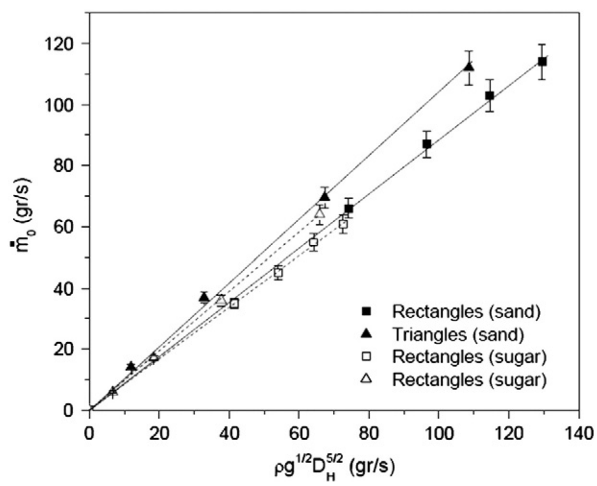

Fig. 4. Plots of the mass flow rate through horizontal slots, $\dot{m}_0$, as a function of $\rho g^{1/2}D_H^{5/2}$. For sand and sugar the data fit the Hagen's law $\dot{m}_0 = a_H\rho g^{1/2}D_H^{5/2}$ very well. For sand: $a_H = 0.89$ for rectangular cross-sections and $a_H = 0.98$ for triangular cross-sections. For sugar: $a_H = 0.84$ for rectangular cross-sections and $a_H = 0.96$ for triangular cross-sections. Error bars are of 4%.

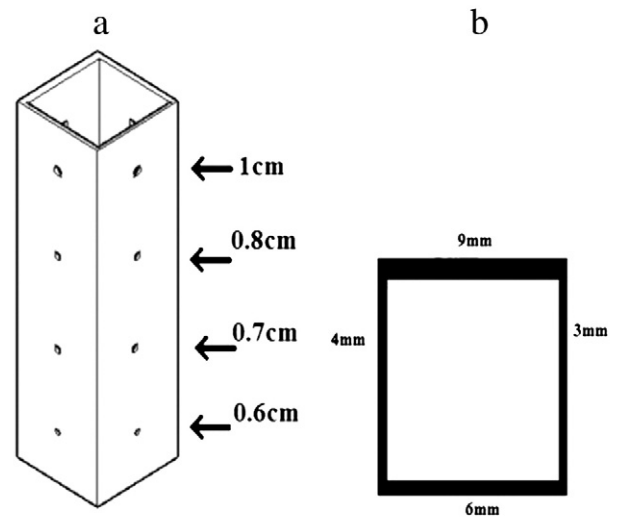

Fig. 5. Depict of the bin used in experiments with lateral circular orifices. (a) Schematic of the bin where the staggered exit holes are shown. Orifices of equal diameter are at the same height. (b) Top view of the silo showing the four different wall thicknesses.

side) wall, respectively. There, it is appreciated through the transparent acrylic, that both granular flows make always approximately the same angle with respect to the horizontal. It has different effects on the outflow because in a thin wall the flow is strongest than in a thick one. These results allow us to propose the next model. In Fig. 7, we depict the lateral view of a region close to an orifice of size D on a vertical wall of a bin when several cases occur. In Fig. 7(c) it is more evident that there is always a natural angle of wall, α, which can be defined as $\alpha = \arctan(D/w)$. Meanwhile, in this same figure it is observed that if there is no flow due to the wall thickness being wide enough, the granular material maintained there will attain its angle of repose, θ_r. Thus, an outflow is kept as long as $\alpha > \theta_r$ (see Fig. 7(a) and (b)) and conversely the outflow should be arrested if $\theta_r > \alpha$. Consequently, the mass flow rate itself must be proportional to $(\alpha - \theta_r)$.

Another important feature to get a general relation for $\dot{m}$ is that the mass flow rate through vertical holes is a fraction of $\dot{m}_0$ (the mass flow rate through bottom holes) [12,13,17,23]. Consequently, the mass flow rate dependence on D and w would be a relation of the form

$$\dot{m} = c\dot{m}_0\left[\arctan\left(\frac{D}{w}\right) - \theta_r\right], \tag{3}$$

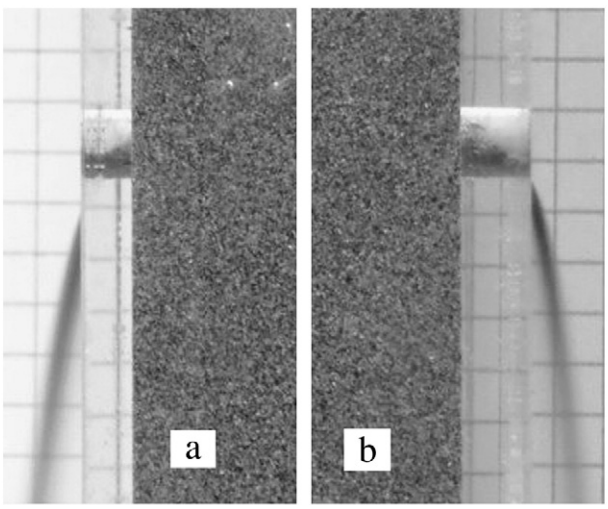

Fig. 6. Snapshots of the region close to the exit hole: (a) thin wall; (b) thick wall.

where c is the corresponding dimensionless discharge coefficient for vertical holes [17]. Notice that now c must depend also on μ_w, the wall friction coefficient, due to the strong interaction among the grains and the inner wall of the bin.

In order to show if Eq. (3) is a correct correlation, for circular orifices, we again performed experiments with sand beach and granulated sugar. In the current experiments we found that the wall friction coefficient among granulated sugar and acrylic was $\mu_w = 0.70 \pm 0.01$ and among sand beach and acrylic it was $\mu_w = 0.80 \pm 0.01$. In Fig. 8 we show the plots of $\dot{m}$ as a function of $\dot{m}_0(\alpha - \theta_r)$, where $\dot{m}_0$ obeys Eq. (1). Here, we used the experimentally measured angles of repose of sand and sugar given in Section 3.1. We measured such angles by using the circular heap method [8].

From plot in Fig. 8 we observe that different sets of data fit very well with straight lines. In our experiments with sand the value of c was $c = 0.23$ and for sugar $c = 0.26$. As in the case of the Hagen's law, these coefficients are different among them because they involve different granular materials and different wall friction coefficients.

Giving all these results, we can conclude that Eq. (3) is a good correlation that fits experimental data very well. Moreover, the previous results give us the confidence to explore other important consequences of Eq. (3). The first one is that Eq. (3) allows us to determine the critical value, w_c, for which the outflow will be arrested, *i.e.*, there is no flow at all when $\dot{m}_0(\alpha - \theta_r) = 0$ or if $\alpha = \theta_r$. Thus, the critical value of w for the arrest, as a function of D and θ_r, is

$$w_c = \frac{D}{\tan\theta_r}. \tag{4}$$

By employing the several values of D used in the experiments with sand and sugar and using their respective angles of repose, we have verified that this relation predicts very accurately the thicknesses of the walls for which the outflow will be arrested.

A second consequence takes into account that $\dot{m}_0 = a\rho g^{1/2}D^{5/2}$, and by using it in Eq. (3), we have found that

$$\dot{m} = c'\rho g^{1/2}D^{5/2}\left[\arctan\left(\frac{D}{w}\right) - \theta_r\right], \tag{5}$$

where the dimensionless parameter c' is $c' = ac$. The use of the respective coefficients yields that $c' = 0.11$ for sand and $c' = 0.12$ for sugar. Eq. (5) is the most explicit formula of the mass flow rate with simultaneous dependence on D and w. An expansion in series of $(D / w) < 1$ transforms Eq. (5) into

$$\dot{m} = c'\rho g^{1/2}\left[\frac{1}{w}D^{7/2} - \theta_r D^{5/2} + O\left(D^{11/2}/w^3\right)\right]. \tag{6}$$

In Eq. (6) the term proportional to $D^{7/2}$ will be dominant if the wall thickness satisfies that $(D / \theta_r) > w$, this last condition is commonly fulfilled when the wall thickness is small with respect to the hole diameter. Due to it, experiments of Bagrintsev and Koshkovskii [11] and Kesava Rao [15,16], where very thin walls were used, have shown that apparently the best correlation among $\dot{m}$ and D takes the form $\dot{m} = c'\rho g^{1/2}D^{7/2}/w$.

3.3. Rectangular and triangular slots

Here, we report experiments with sand and sugar crossing rectangular and triangular slots on the vertical walls of bins under the aforementioned conditions. Applying a similar analysis as above, in Fig. 9 plots of the measured mass flow rates, as a function of $\dot{m}_0(\alpha - \theta_r)$, are shown for vertical slots. Now $\dot{m}_0$ is given by Eq. (2). For rectangular slots, the angle of wall here is given by $\alpha = \arctan(f / n)$ and for triangular slots $\alpha = \arctan(h / w)$, where the height of an equilateral triangle, h,

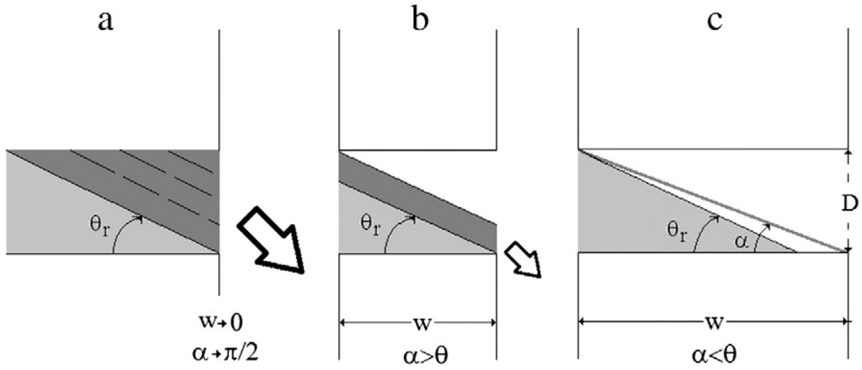

Fig. 7. Schematic lateral view of the exit hole in a vertical bin when granular material crosses it. In panel a the wall thickness is close to zero, consequently, $\alpha \to \pi / 2$ and the granular material in the dark-gray zone, characterized by the angle $\pi / 2 - \theta_r$ will cross the orifice meanwhile the granular material located at the light-gray zone will be maintained therein. In panel b the wall thickness yields the geometric condition $\alpha > \theta_r$ and the granular outflow occurs in the dark-gray zone which is proportional to $(\alpha - \theta_r)$. Finally, in panel c the wall thickness produces the condition $\alpha < \theta_r$, whence there is no flow. The granular flow is arrested in such a form that the slope of the granular material in the exit hole is featured by the angle of repose.

of length l, is $h = \sqrt{3}l/2$. Experiments with both materials yield data that fit relations of the form

$$\dot{m} = c\dot{m}_0[\alpha - \theta_r], \tag{7}$$

where for rectangular slots $c = 0.30$, and for triangular slots $c = 0.20$, in the case of sand. For sugar, we found that $c = 0.29$, for rectangles, and $c = 0.19$, for triangles. For slots, the explicit dependence of$\dot{m}$on D_H is given by

$$\dot{m} = c' \rho g^{1/2} D_H^{5/2}[\alpha - \theta_r], \tag{8}$$

whence c' is now $c' = a_H c$. From experiments with sand $c' = 0.27$, for rectangular slots, and $c' = 0.20$, for triangular slots. For sugar $c' = 0.24$, for rectangular slots, and $c' = 0.18$, for triangular slots.

Finally, the corresponding formulas to quantify the critical thickness to arrest the granular flow are: $w_c = f / \tan\theta_r$ for rectangular slots and $w_c = h / \tan\theta_r$ for triangular slots. It means that the effective hydraulic diameters for slots will not be involved in the estimation of w_c.

4. Conclusions

In this work we studied experimentally the problems of the mass flow rate of granular material through circular orifices and rectangular and triangular slots on vertical walls of bins as a small step toward understanding grains flowability. Specifically, we have studied, simultaneously, the dependence of$\dot{m}$on the diameter of the orifice and the wall thickness by using well characterized sand beach and granulated sugar. To our knowledge, this is the first time that a systematic experimental and theoretical study of the effect of the wall thickness on the mass flow rate has been done. Our experiments performed with sand beach and granulated sugar show that Eqs. (3) and (7) describe very well the influence of w and D (or D_H) on the mass flow rate of grains crossing circular orifices and rectangular and triangular slots, and they can be considered as general formulas including the dependence on both quantities. In such equations the angle of wall and the angle of repose are fundamental to describe the occurrence of the outflow of grains and its arrest. Moreover, when the wall thickness is small with respect to the orifice diameters, the explicit dependence on D yields that essentially $\dot{m} \sim D^{7/2}$; close results were previously reported in some experiments of vertical orifices [11,15,16] but our result is obtained directly from our current model. For slots in bins with very thin walls$\dot{m}$depends on the geometrical parameters in a slightly more complex way than the simple one given in the context of circular orifices.

Some studies have shown that the discharge rates from bottom exits decrease with the increase in particle size [7]. In such a case, instead of the term $D^{5/2}$ in Eq. (1), it was introduced the annular zone effect through the term $(D - kd_g)^{5/2}$ ($(D_H - kd_g)^{5/2}$ for slots) where k is a

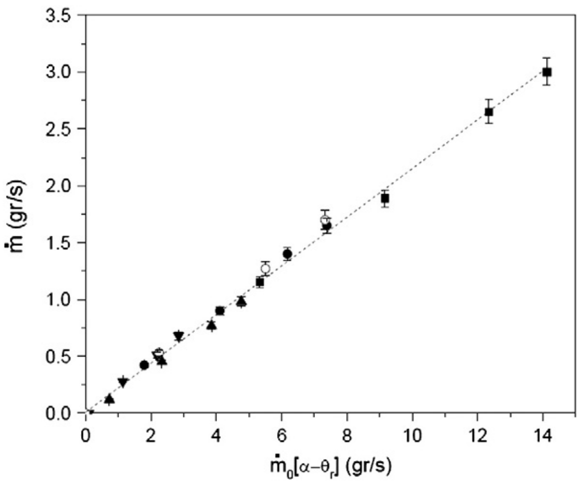

Fig. 8. Plot of the mass flow rate through circular holes on side walls$\dot{m}$as a function of $\dot{m}_0[\alpha - \theta_r]$. $\dot{m}_0$ was taken from plot of Fig. 3. Solid symbols are for sand and open symbols are for sugar. Data fit accurately Eq. (3) with $c = 0.23$ for sand and $c = 0.26$ for sugar. Error bars are 4%.

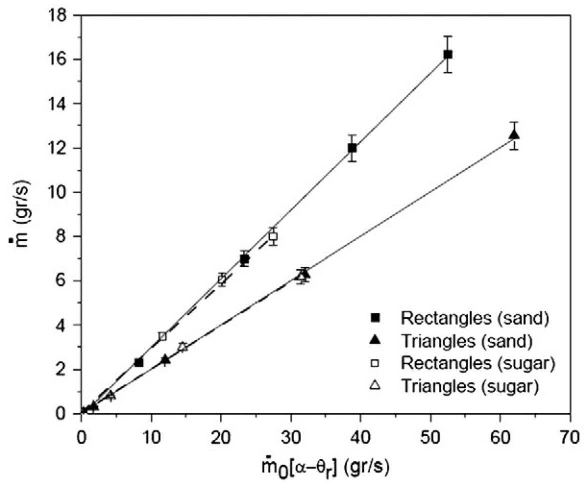

Fig. 9. Plot of the mass flow rate through triangular and rectangular orifices on side walls, $\dot{m}$, as of function of $\dot{m}_0[\alpha - \theta_r]$. $\dot{m}_0$ was taken from plot of Fig. 4. Solid symbols are for sand and open symbols are for sugar. Fits yield, for rectangular slots $c = 0.30$, and for triangular slots $c = 0.20$, in the case of sand. For sugar, we found that $c = 0.29$, for rectangles, and $c = 0.19$, for triangles. Error bars are 4%.

dimensionless constant); however, the use of this type of correlation for vertical orifices requires the performance of more studies in order to have a general formula valid in a wide range of practical configurations. Work along these lines is now in progress.

Some results of the current study could be of importance in the aeration of coarse grains in bins because in this case air flows easily through voids among grains. Efficient processes of aeration involve high permeabilities [19] and several staggered orifices along the vertical wall of the bin itself [24–26]. Finally, in a similar context, the Agricultural and Food Engineering Technologies Service (AGST) of the Food and Agriculture Organization of the United Nations, in a recent document [27], has reported the use of the household metal silos, with lateral exit holes, as a key post-harvest technology in the fight against hunger and for food security. It is clear that our mass flow rate formulas are also useful to estimate the mass flow rate in these type of systems.

Acknowledgments

The authors acknowledge Prof. K. Kesava Rao for his useful observations during the realization of this work. They also acknowledge A. Serrano, I. Saldivar and S. Alvarez for their aid in the making of experiments. Finally, we acknowledge partial support of projects 20131821 and 20130651 from IPN and project 146735 from the SENER-CONACyT fund.

References

[1] A. Garcimartín, C. Mankoc, A. Janda, R. Arevalo, J.M. Pastor, I. Zuriguel, D. Maza, Flow and jamming of granular matter through an orifice, in: C. Appert-Rolland, et al., (Eds.), Traffic and Granular Flow' 07, 2009, pp. 471–486.
[2] J. Andrade, C. Treviño, A. Medina, Experimental evidence of density waves in two-dimensional bins, Phys. Lett. A 223 (1996) 105–111.
[3] J.K. Prescot, R.A. Barnum, On powder flowability, Pharma. Technol. 24 (10) (2000) 60–84.
[4] G.H.L. Hagen, Aber den Druck und die Bewegung des trocknen Sandes, Bericht uber die zur Bekanntmachung geeigneten Verhandlungen der Koniglich Preussischen Akademie der Wissenschaften zu Berlin, 35, 1852, pp. 35–42.
[5] C. Mankoc, A. Janda, R. Arevalo, J.M. Pastor, I. Zuriguel, A. Garcimartin, D. Maza, The flow rate of granular materials through an orifice, Granul. Matter 9 (2007) 407–414.
[6] C.R. Woodcock, J.S. Mason, Bulk Solids Handling: An Introduction to the Practice and Technology, Blackie and Son Ltd., London, 1987. 47–63 (Chap. 2).
[7] W.A. Beverloo, H.A. Leniger, J. van de Velde, The flow of granular solids through orifices, Chem. Eng. Sci. 15 (1961) 260–269.
[8] K. Wieghardt, Experiments in granular flow, Annu. Rev. Fluid Mech. 7 (1975) 89–114.
[9] I.I. Kotchanova, Experimental and theoretical investigation on the discharge of granular materials from bins, Powder Technol. 4 (1970/71) 32–37.
[10] F.M. White, Viscous Fluid Flow, Third edition McGraw-Hill, New York, 2006. 116.
[11] I.I. Bagrintsev, S.S. Koshkovskii, Investigation of the outflow of granular materials through openings in the wall of a vertical cylindrical tube, J. Chem. Pet. Eng. 6 (1977) 503–505.
[12] C.E. Davies, J. Foye, Flow of granular material through vertical slots, Trans. Inst. Chem. Eng. 69 (1991) 369–373.
[13] C.S. Chang, H.H. Converse, J.L. Steele, Flow rates of grain through various shapes of vertical and horizontal orifices, Trans. Am. Soc. Agric. Eng. 34 (1991) 1789–1796.
[14] C.E. Davies, M. Desai, Blockage in vertical slots: experimental measurement of minimum slot width for a variety of granular materials, Powder Technol. 183 (2008) 436–440.
[15] S. Choudary, K. Kesava Rao, Indian Academy of Science, Project Report, 2006.
[16] S.A. Kumar, K. Kesava Rao, Indian Academy of Science, Project Report, 2006.
[17] H.G. Sheldon, D.J. Durian, Granular discharge and clogging for tilted hoppers, Granul. Matter 12 (2010) 579–585.
[18] See, for instance, Ref. [6], Chap. 2, pp. 77–81.
[19] S.-S. Hsiau, C.-C. Hsu, J. Smid, Discharge of fine silica sands in a silo, Phys. Fluids 22 (2010) 043306.
[20] C.A. Davy, M.D. Bolton, N.A. Fleck, The shearing behaviour of a sugar aggregate, Acta Mater. 52 (2004) 3587–3601.
[21] E. Deming, A.L. Mehring, The gravitational flow of fertilizers and other comminuted solids, Ind. Eng. Chem. 21 (1929) 661–665.
[22] T. Le Pennec, M. Ammi, J.C. Messager, B. Truffin, D. Bideau, J. Garnier, Powder Technol. 85 (1995) 279–281.
[23] F.C. Franklin, L.N. Johanson, Flow of granular material through a circular orifice, Chem. Eng. Sci. 4 (1955) 119–129.
[24] G.R. Thorpe, Modelling ecosystems in ventilated conical bottomed farm grain silos, Ecol. Model. 94 (1997) 255–286.
[25] I.P. Bibby, S.T. Conyers, Numerical simulations of gas exchange in leaky grain silos, using measured boundary conditions, J. Stored Prod. Res. 34 (1998) 217–229.
[26] D. de Carvalho Lopes, J.H. Martins, E. de Castro Melo, P.M. de Marros Monteiro, Aeration simulation of stored grain under variable air ambient conditions, Postharvest Biol. Technol. 42 (2006) 115–120.
[27] Household Metal Silos, Agricultural and Food Engineering Technologies Service, Food and Agriculture Organization of the United Nations, Viale delle Terme di Caracalla, 00153, Rome, Italy, 2008.