A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

OFFPRINT

Super free fall of an inviscid liquid through interconnected vertical pipes

C. Treviño, S. Peralta, A. Torres and A. Medina

EPL, **112** (2015) 14002

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

AN INVITATION TO SUBMIT YOUR WORK

www.epljournal.org

The Editorial Board invites you to submit your letters to EPL

EPL is a leading international journal publishing original, innovative Letters in all areas of physics, ranging from condensed matter topics and interdisciplinary research to astrophysics, geophysics, plasma and fusion sciences, including those with application potential.

The high profile of the journal combined with the excellent scientific quality of the articles ensures that EPL is an essential resource for its worldwide audience. EPL offers authors global visibility and a great opportunity to share their work with others across the whole of the physics community.

Run by active scientists, for scientists

EPL is reviewed by scientists for scientists, to serve and support the international scientific community. The Editorial Board is a team of active research scientists with an expert understanding of the needs of both authors and researchers.

www.epljournal.org

epl A Letters Journal Exploring the Frontiers of Physics

www.epljournal.org

OVER **560,000** full text downloads in 2013

24 DAYS average accept to online publication in 2013

10,755 citations in 2013

"We greatly appreciate the efficient, professional and rapid processing of our paper by your team."

Cong Lin
Shanghai University

Six good reasons to publish with EPL

We want to work with you to gain recognition for your research through worldwide visibility and high citations. As an EPL author, you will benefit from:

1. **Quality** – The 50+ Co-editors, who are experts in their field, oversee the entire peer-review process, from selection of the referees to making all final acceptance decisions.
2. **Convenience** – Easy to access compilations of recent articles in specific narrow fields available on the website.
3. **Speed of processing** – We aim to provide you with a quick and efficient service; the median time from submission to online publication is under 100 days.
4. **High visibility** – Strong promotion and visibility through material available at over 300 events annually, distributed via e-mail, and targeted mailshot newsletters.
5. **International reach** – Over 2600 institutions have access to EPL, enabling your work to be read by your peers in 90 countries.
6. **Open access** – Articles are offered open access for a one-off author payment; green open access on all others with a 12-month embargo.

Details on preparing, submitting and tracking the progress of your manuscript from submission to acceptance are available on the EPL submission website **www.epletters.net**.

If you would like further information about our author service or EPL in general, please visit **www.epljournal.org** or e-mail us at **info@epljournal.org**.

EPL is published in partnership with:

EPS Società Italiana di Fisica edp sciences IOP Publishing

European Physical Society | Società Italiana di Fisica | EDP Sciences | IOP Publishing

A Letters Journal Exploring the Frontiers of Physics

October 2015

EPL, **112** (2015) 14002
doi: 10.1209/0295-5075/112/14002

www.epljournal.org

Super free fall of an inviscid liquid through interconnected vertical pipes

C. Treviño[1], S. Peralta[2], A. Torres[2] and A. Medina[2]

[1] *Unidad Multidisciplinaria de Docencia e Investigación Sisal Facultad de Ciencias, UNAM Puerto de Abrigo S/N, Sisal, Yucatán 97355, México*
[2] *SEPI ESIME Azcapotzalco, Instituto Politénico Nacional - Av. de las Granjas No. 682, Col. Sta. Catarina, C.P. 02250, Mexico D.F., México*

received 18 June 2015; accepted in final form 1 October 2015
published online 19 October 2015

PACS 47.60.Dx – Flows in ducts and channels
PACS 47.10.-g – General theory in fluid dynamics
PACS 47.80.Jk – Flow visualization and imaging

Abstract – In this work the super free fall motion of the interface of a liquid column flowing downwards, suddenly from the rest in a system of vertical concentrically interconnected tubes, is studied theoretically, using a one-dimensional model in the inviscid regime. The ratio of the surface area of the upper tube to that of the lower one, A_1/A_2, is assumed to be smaller than unity. Experiments show a good agreement with the main results of the developed theory.

Introduction. – Super free fall, *i.e.*, motion with acceleration higher than the gravity acceleration, g, has been recently reported as typical of gravity-induced fluid flows in geometries where the cross-section of vertical pipes increases smoothly and continuously along the flow direction [1,2]. The condition of liquid with very low viscosity also is critical in order to minimize the viscous effects such as the viscous dissipation and the viscous boundary layer.

In this work another configuration, the interconnected concentric pipes, is analyzed, where a vertical upper tube of cross-sectional area, A_1, is joined at a specific location to another pipe of larger area, A_2, with $A_2 > A_1$. The consideration of a sudden change in the pipe diameter is regarded here to contrast it with the case where the pipe cross-section increases smoothly (in pipes with larger expansion rates the growth and detachment of the viscous boundary layers along the tube walls may occur in experiments, even with low-viscosity fluids, and make the phenomenon disappear in practice) [1]. On the contrary, in pipes with sudden enlargement and very low-viscosity fluids, the uniform upstream flow develops a stream of discharge into the wide section in the form of a straight jet and motion irregularities, at the side of the jet, which rapidly are mixed with the jet and finally the stream again is approximately uniform [3].

By the way, problems where a liquid's super acceleration occurs may be of interest in the study of ocean surface waves, for instance, where the mechanisms of energy interchange are very complex [4–6]. Super free fall in liquids also resembles the super accelerated motion of the tip of a chain under free fall [7,8].

Our experimental studies allow to show that the inertial instability (nipple) growing in the upper free surface in conical pipes [1,2] does not appear in the case of super free fall in interconnected pipes. Details of this point will be given later.

The goal of this work is to study the super free fall through interconnected pipes and to contrast the main properties of this super accelerated flow against the properties of the super accelerated motion in conical pipes. To reach this purpose, in the first section the problem formulation for the inviscid one-dimensional flow is written down and a global nondimensional nonlinear ordinary differential equation is obtained for the free surface position as a function of two geometrical parameters. In the second section the numerical results are presented. In the third section a set of experiments are discussed and compared with the numerical results and finally, in the last section the main conclusions of this work are discussed.

Problem formulation and analysis. – When an ideal liquid is contained in a vertical cylindrical pipe and suddenly the lower part of it is opened, the gravity will accelerate the liquid up to this flow attains an acceleration of magnitude g.

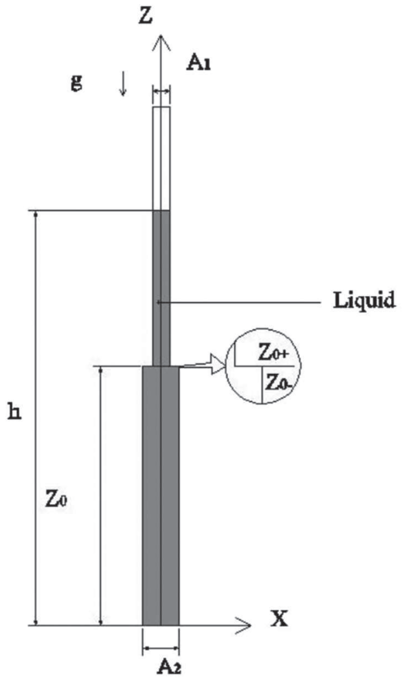

Fig. 1: Scheme of concentric interconnected tubes. The z-axis points upward and is opposite to the gravity acceleration vector.

A more interesting case occurs when two vertical pipes are interconnected, as depicted in fig. 1. There, a cylindrical tube with transversal area A_1, is connected at $z = z_0$ with another tube with transversal area A_2, with the assumption that $A_2 > A_1$ and the initial liquid's height, h_0, exceeds the point of union, that is $h_0 > z_0$. If the bottom exit is open suddenly at time $t = 0$, it is also assumed that a quasi–one-dimensional flow develops, with the momentum equation given by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}z} - g, \tag{1}$$

where u is the fluid velocity, ρ is the fluid density and p is the pressure. Moreover, the mass conservation equation can be written as $A_1u_1(t) = A_2u_2(t)$ or $u_2(t) = \alpha u_1(t)$, where $\alpha = A_1/A_2 < 1$. If the pressure is $p = 0$ at $z = 0$ and $z = h$, and eq. (1) is integrated from $z = z_{0+}$ (see fig. 1) up to $z = h$, it is found that

$$\ddot{h}\left(h - z_{0+}\right) = \frac{1}{\rho}p_{z_{0+}} - g\left(h - z_{0+}\right), \tag{2}$$

where the primes denote time derivatives.

Similarly, upon integration from $z = 0$ up to $z = z_{0-}$, eq. (1) gives

$$\alpha\ddot{h}z_{0-} = -\frac{1}{\rho}p_{z_{0-}} - gz_{0-}. \tag{3}$$

Considering that $z_0 = z_{0+} = z_{0-}$, the use of eqs. (2) and (3) yields the relation

$$\frac{1}{\rho}(p_{z_{0+}} - p_{z_{0-}}) = \ddot{h}\left[h - (1-\alpha)z_0\right] + gh. \tag{4}$$

The jump condition for pressures at the joining point is given by the Bernoulli theorem

$$\frac{1}{\rho}(p_{z_{0+}} - p_{z_{0-}}) = -\frac{1}{2}(\dot{h})^2(1 - \alpha^2 - K), \tag{5}$$

where K is the head loss parameter due to the sudden expansion [3,9], with $K \simeq (1-\alpha)^2$.

It is important to comment that eq. (5) can be obtained by using the momentum integral theorem to estimate the total stress in a control-volume close to the zone of sudden enlargement $z = z_0$, and it yields [3]

$$p_{z_{0-}} = p_{z_{0+}} + \rho u_2\left(u_1 - u_2\right) = p_{z_{0+}} + \rho\alpha(\dot{h})^2(1-\alpha). \tag{6}$$

By the way, for a control-volume embracing zones far from $z = z_0$, where the flow is everywhere steady, the Bernoulli equation lets us find that [3]

$$p_{z_{0-}} = p_{z_{0+}} + \frac{1}{2}\rho\left(u_1^2 - u_2^2\right) = p_{z_{0+}} + \frac{1}{2}\rho(\dot{h})^2(1-\alpha^2). \tag{7}$$

The addition of the term $(\rho/2)\,(\dot{h})^2(1-\alpha)^2 \simeq (\rho/2)\,(\dot{h})^2K$ to eq. (6) gives eq. (7). Consequently, the added term can be seen as due to an eddying mixing flow that produces a pressure fall in the Bernoulli equation (7); a simple rearranging of terms gives eq. (5).

Finally, from eqs. (4) and (5), the evolution equation for h takes the form

$$\ddot{h}\left[h - (1-\alpha)z_0\right] + \frac{1}{2}(\dot{h})^2(1 - \alpha^2 - K) + gh = 0, \tag{8}$$

which has to be solved with the initial conditions $h(0) = h_0$ and $\dot{h}(0) = 0$.

At short times, when the motion starts, $\dot{h}(0) = 0$, and thus the initial acceleration is

$$\ddot{h}(0) = -\frac{gh_0}{h_0 - (1-\alpha)z_0}, \tag{9}$$

which gives larger initial accelerations than g for values of $\alpha < 1$. However, if $\alpha = 1$ the free fall case is achieved.

To get the complete history of the motion of the upper free surface, the solution of eq. (8) has to be obtained. Using the following nondimensional variables:

$$H = \frac{h}{h_0}, \quad \tau = \sqrt{\frac{g}{h_0}}t, \quad \beta = \frac{z_0}{h_0}, \tag{10}$$

the nondimensional form of the evolution equation (8) reduces to

$$\ddot{H}\left[H - (1-\alpha)\,\beta\right] + \frac{1}{2}\left[\dot{H}\right]^2\left(1 - \alpha^2 - K\right) + H = 0. \tag{11}$$

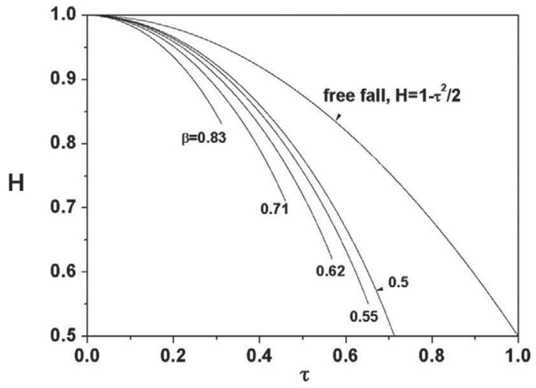

Fig. 2: Plot of the nondimensional position, H, of the upper free surface, as a function of time. Several values of $\beta = z_0/h_0$ (levels of filling) were considered meanwhile the ratio of areas is assumed as $\alpha = 0.178$.

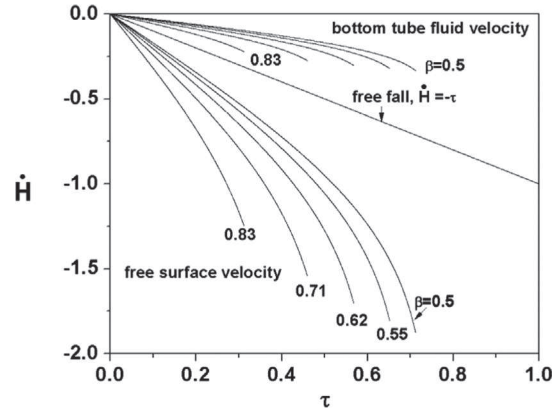

Fig. 3: Dimensionless plot of the velocity of the free surface as a function of time. Here $\alpha = 0.178$ and same values of β, as in fig. 2, were assumed.

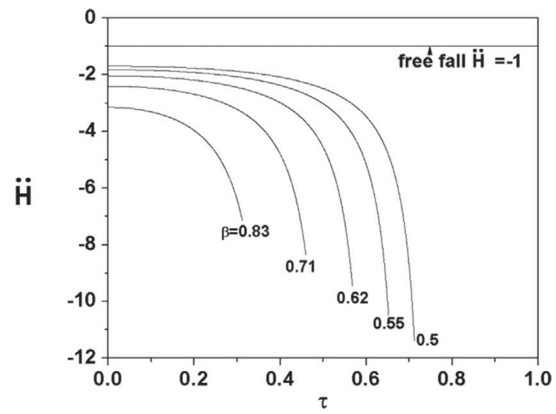

Fig. 4: Plot of the nondimensional acceleration as a function of time for several values of β. As in fig. 3, $\alpha = 0.178$.

A first integration can be readily obtained from the autonomous eq. (11), giving

$$\frac{\mathrm{d}T}{\mathrm{d}H}\left[H-(1-\alpha)\beta\right]+T(1-\alpha^2-K)+H=0, \qquad (12)$$

where T is the nondimensional kinetic energy, $T = (\dot{H})^2/2$. The initial condition now can be written as $T(1) = 0$, valid in the range $H > \beta$. For the particular case $\alpha = 1$ (free fall), the solution of eq. (12) gives $T = 1 - H$, *i.e.* the potential energy transforms linearly into kinetic energy. When $\alpha \neq 1$ the transformation of potential to kinetic energy is nonlinear. The analytical solution of eq. (12) has of the form

$$T=-\frac{H}{B}+\frac{(H-A)}{B(B+1)}+\frac{(1-A)^B}{B(H-A)^B}-\frac{(1-A)^{B+1}}{B(B+1)(H-A)^B}, \qquad (13)$$

where $A = (1-\alpha)\beta$ and $B = 1-\alpha^2-K$, this confirms the nonlinear dependence of T on H.

At this point, it is important to highlight that in their study of the super free fall of liquids in tapered pipes, Villermaux and Pomeau [1] have shown that the existence of a central nipple (Rayleigh-Taylor instability), in the free surface, is strongly linked to the necessary increasing of the pipe's radius ($\partial_z R \neq 0$, where $R(z)$ is the radius of the conical pipe and z is the axial coordinate, see eq. (4.11) in [1]). In the current work, $\partial_z R = 0$ for both straight pipes, and $\partial_z R \to \infty$ at $z_0 = z_{0+} = z_{0-}$. Hence, the main condition for the possible existence of a central nipple is not satisfied and, consequently, a central nipple could not be expected. Experiments will reinforce this result.

Numerical results. – The numerical solutions of eq. (11) were obtained for $\alpha = 0.178$ under the initial conditions $H = \dot{H} = 0$ at $\tau = 0$. This value of α will be useful for comparison with experiments. In fig. 2 plots of the free surface nondimensional height H against τ are shown for several values of β. An important result is that as the value of β increases, the function H decreases faster with the nondimensional time. This means that when the upper pipe is filled up to a small height, the acceleration of the upper free surface increases. This result is in apparent contradiction to that occurring in the conical pipes [1,2], because if such a pipe is brimful the super acceleration is stronger. However, in this latter case the value of α is a function of β and decreases as β increases. Also, in the same figure, the free fall case given by $H = 1 - \tau^2/2$, has been plotted.

In fig. 3 the nondimensional free surface velocity, $\dot{H}$ and the fluid velocity at the lower tube, $\alpha\dot{H}$, are plotted as functions of time, for the same values of β employed in fig. 2. The free fall velocity, $\dot{H} = -\tau$, is again shown to compare the flow behavior. The free surface moves faster than the free fall case, whereas the fluid leaves the bottom tube with lower velocity than the free fall case. Higher velocities (accelerations) are achieved for larger values of β for fixed values of α. The nondimensional free surface acceleration, $\ddot{H}$, as function of time, is plotted in fig. 4, for the same set of parameters α and β as before. The free fall case, $\ddot{H} = -1$, is also plotted. Finally, after solving numerically eq. (12), the nondimensional kinetic

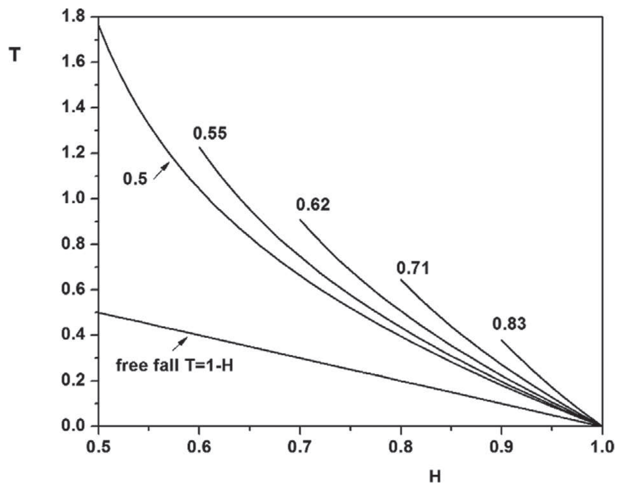

Fig. 5: Nondimensional plots of kinetic energy, T, as a function of the potential energy, H for several values of β. The numerical and analytical (eq. (13)) solutions coincide exactly.

Fig. 6: Snapshots of the gravity-induced flow in concentric pipes. In this case $\alpha = 0.178$ and $\beta = 0.833$. Pipes were filled with ethanol. Times from left to right are $t = 0$, 11/500, 16/500 and 27/500 s.

energy T is plotted in fig. 5 as a function of the nondimensional free surface height or potential energy, H, for the same parametric set. The analytical solution given by eq. (13) follows exactly the same behavior. The free fall case, $T = 1 - H$, is also included.

Experiments. – In order to test the usefulness of the simplified one-dimensional theory, a number of experiments were designed and carried out. In a first case, two acrylic tubes were joined in a concentric form. The upper tube was: length = 0.5 m, diameter = 0.0184 m and the lower tube was: length = 0.5 m, diameter = 0.0435 m. The system of vertical interconnected pipes was filled with ethanol (density $\rho = 810\,\text{kg/m}^3$, kinematic viscosity $\nu = 1.52 \times 10^{-6}\,\text{m}^2/\text{s}$).

Initially, a piston has been used to suddenly open the bottom exit. The resulting flow was affected by an effect similar to that when a cat laps [10]. Instead of opening by using this procedure, an elastic sheet has been employed to seal the bottom exit and it was suddenly broken to start the flow. Incidentally, this method was used by Bernoulli [11] near three centuries ago to induce

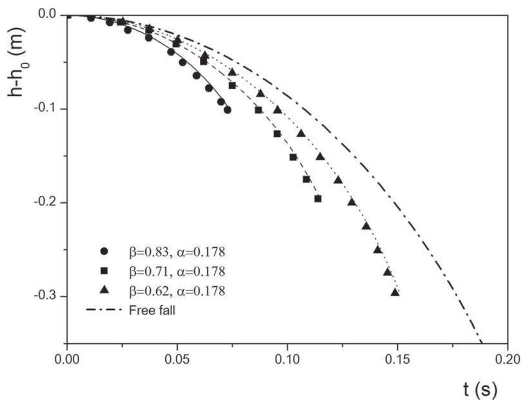

Fig. 7: Plot of the positions of the upper free surface, $h - h_0$, as a function of time. Data were taken from experiment with values $\beta = 0.83$, $\beta = 0.71$, $\beta = 0.62$ for $\alpha = 0.178$. Curves corresponds to the numerical solutions.

flows in pipes. Each experiment was video recorded with a high-speed camera (1/500 fps) and the relative position of the upper free surface was measured as a function of time. Experiments were performed using the following values of the dimensionless parameters: $\alpha = 0.178$ and $\beta = 0.83, 0.71, 0.62$.

In fig. 6 a collection of snapshots of the falling flow for the case with $\alpha = 0.178$ and $\beta = 0.833$ are shown. It is apparent from these pictures that the nipple was not formed. Figure 7 shows (in physical variables) the comparison between the experimental data of the flow shown in fig. 6 and the numerical solution for the same parametric set of the experiment. The agreement is excellent, which supports the validity of using an over-simplified one-dimensional theoretical approach.

Discussion and conclusions. – In this work a theoretical and experimental work has been undertaken to study the super free fall characteristics in an arrangement of concentric tubes. It is possible to highlight several important characteristics of the motion of the free surface in this system. a) The free surface can achieve persistent accelerations several times larger than g; b) the acceleration is larger for smaller levels of filling in the upper pipe and c) a nipple was not found at the interface between both tubes. A possible reason for this is that in the employed configuration in this work the expansion of the pipe is sudden and localized. In the analysis given by Villermaux and Pomeau [1] the existence of the nipple is linked to a smooth and continuous aperture of the pipe and this condition is not fulfilled by the interconnected pipes.

REFERENCES

[1] Villermaux E. and Pomeau Y., *J. Fluid Mech.*, **642** (2010) 147.

[2] Torres A., Medina A., Higuera F. J. and Weidman P. D., *J. Fluid Mech.*, **723** (2013) 653.

[3] Batchelor G. K., *An Introduction to Fluid Dynamics* (Cambridge University Press, Cambridge) 2000.
[4] Garner D. M., *Dtsch. Hydrogr. Z.*, **22** (1969) 163.
[5] Phillips O. M., *Dynamics of the Upper Ocean* (Cambridge University Press, Cambridge) 1980.
[6] Mitsuyasu H., *J. Oceanogr.*, **58** (2002) 109.
[7] Calkin M. G. and March R. H., *Am. J. Phys.*, **57** (1989) 154.
[8] Tomaszewski W., Pieranski P. and Geminard J.-C., *Am. J. Phys.*, **74** (2006) 776.
[9] Worster G., *Understanding Fluid Flow* (Cambridge University Press, Cambridge) 2009.
[10] Reis P. M., Jung S., Aristoff J. M. and Stoker R., *Science*, **330** (2010) 1231.
[11] Bernoulli D., *Hydrodynamica* (Johann Reinhold, Strassburg) 1738.