Thermal nonlinear oscillator in mixed convection

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A detailed numerical simulation is carried out for transient laminar flow opposing mixed convection in a downward vertical channel flow with both walls suddenly subjected to isothermal heat sources over a finite portion of the channel walls, by solving the unsteady two-dimensional Navier-Stokes and energy equations. The dynamical behavior of the system is influenced by, in addition to the geometrical parameters, three nondimensional parameters: the Reynolds, Richardson, and Prandtl numbers. Numerical experiments were performed for fixed values of the geometrical parameters, the Reynolds number (Re = 100) and the Prandtl number (Pr = 7). With variation in the value of the buoyancy (Richardson number), the nonlinear dynamical response of the system can reach (i) a stationary solution, (ii) a local and then a global periodic solution where the system executes self-sustained relaxation oscillations, or (iii) a solution in which the relaxation oscillation is destroyed leading to a chaotic state. In this study, bifurcations between different states, phase-space plots of the self-oscillatory system, characteristic times of temperature oscillations, and an exact description of the oscillations are presented quantitatively for a range of values of the buoyancy parameter. The results include the effects of the Reynolds and Prandtl numbers on the evolution of the different transitions.

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I. INTRODUCTION

Mixed convection is defined as a heat transfer situation where both natural and forced convection mechanisms interact. In particular, the oscillatory behavior in mixed convection flows is of great interest because of its rich dynamical features and useful results for applied problems. Chang and Lin [1] studied steady laminar and transient oscillatory mixed convection in a symmetrically heated vertical plane channel subjected to an opposing buoyancy, assuming a fully developed velocity profile at the inlet and discrete heat sources that are maintained at uniform and equal heat fluxes. The authors pointed out that an oscillatory flow with a single fundamental frequency is found when the buoyancy parameter, or Richardson number, exceeds a critical value. Lin *et al.* [2] investigated numerically the detailed flow and thermal characteristics in transient laminar flow opposing mixed convection in a vertical plane channel subjected to a symmetrical heat input. Their results show that at high opposing buoyancy, sudden flow asymmetry and oscillation occur simultaneously in an early steady flow after the initial transient. Evans and Greif [3] showed the strong effects of buoyancy, even for small temperature differences, on the downward flow of nitrogen in a partially heated tall vertical channel and reported time-dependent oscillations, including periodic flow reversals along the channel walls. Martínez-Suástegui and Treviño [4,5] investigated the transient laminar mixed convection in an asymmetrically and differentially heated vertical channel of finite length subjected to an opposing buoyancy. Their results show that a final steady or oscillatory flow response is obtained depending on the values of the Reynolds and Richardson numbers, and that the critical value of the buoyancy strength between the two regimes strongly depends on the value of the Reynolds number. Stability analyses in mixed convection flows have been developed through recent years and their results provide further insight into the instability mechanisms present in such situations and give quantitative information about the defining and critical parameters involved. Guillet *et al.* [6] considered the problem of laminar-assisted mixed convection flow between parallel, vertical, and uniformly heated plates where the governing dimensionless parameters are the Prandtl, Rayleigh, and Reynolds numbers. By use of a method based on the center manifold theorem, the authors proved that there is a pitchfork bifurcation in the system for a critical value of the Rayleigh number. Chen and Chung [7,8] studied the stability of a differentially heated vertical channel for various Prandtl numbers and showed that both the Prandtl number and the Reynolds number have very important effects on the instability mechanism for high Prandtl number fluids. Suslov and Paolucci [9] studied the stability of mixed convection flow in a tall vertical channel under non-Boussinesq conditions and showed that the stability characteristics, such as the critical Grashof number and the disturbance wave speed, depend strongly on the temperature difference when fluid properties are allowed to vary. Bera and Khalili [10] numerically studied the impact of permeability on the stability of a buoyancy-opposed fully developed basic flow in a vertical channel. Daniels [11] studied the stationary instability of the convective flow between differentially heated vertical planes and determined the subsequent structure of the neutral curve for stationary disturbances.

Although the oscillatory behavior of Navier-Stokes-type systems in mixed convection has received relatively little attention, it is known that these flows can exhibit interesting

