



## Asymptotic analysis for the conjugate heat transfer problem in an electro-osmotic flow with temperature-dependent properties in a capillary

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### ABSTRACT

In this work, the conjugate heat transfer process in an electro-osmotic flow of a Newtonian liquid is studied asymptotically. The analysis includes Joule heating effects by taking into account the temperature dependent viscosity and electrical conductivity of the electrolyte solution and assuming finite thermal conductivity of the capillary wall. Due to Joule heating effects, temperature gradients in the liquid make the fluid properties change within the capillary, altering the electric potential and flow fields. The dimensionless temperature profiles in the fluid and the capillary wall are obtained as function of the dimensionless parameters involved in the analysis, and the interactions between the coupled continuity, momentum, thermal energy, and potential electric equations are examined in detail. Results show that the Joule heating induces a pressure gradient along the capillary, which in turn modifies the normal plug-like electroosmotic velocity profiles. In addition, it is pointed out that, depending on the values of the dimensionless parameters, the modified velocity profiles can induce positive or negative pressure gradients at the inlet or outlet of the capillary.

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### 1. Introduction

In forced convection heat transfer problems involving channels of conventional size, the wall thickness is usually very small compared to the hydraulic diameter. In these cases, the longitudinal heat conduction through the walls is usually neglected. On the other hand, in micro-scale thermal devices, the area of the transverse section of the solid material is comparable to the area of the cross-section of the channel, and heat conduction effects through the solid wall have to be considered. In this sense, there have been some attempts to elucidate scaling effects that are of great importance in micro-flow devices [1–3], from which the following can be identified as crucial: (i) entrance effects, conjugate heat transfer (relatively thick walls) (ii) electric double layer (EDL) effects, (iii) temperature dependent properties, and (iv) wall roughness. Multiple investigations have studied the effects of Joule heating in electro-osmotic flows. Analytical solutions for steady electro-osmotic flows with Joule heating and constant thermo-physical properties were developed in [4–8]. Chao et al. [9] presented numerical results for the Joule heating effect of electro-osmosis in a finite-length microchannel and pointed out that the longitudinal temperature variation induces a pressure gradient

that changes the behavior of the electric field in the microchannel. Xuan et al. [10] examined numerically and experimentally the electro-osmotic flow with Joule heating effects and reported velocity perturbations due to the induced pressure gradients resulting from axial variations of temperature. In [11], Xuang reviewed the recent progress on the topic of Joule heating and its effect in electrokinetic flow, concluding that conjugate heat transfer problems for electro-osmotic flow must include temperature dependent properties for viscosity and electrical conductivity of the fluid. The Joule heating effects of electro-osmosis in liquid-solid coupled microchannels, including the feedback effects of temperature variation on liquid properties were widely studied in [12–15]. Xuan et al. [16] investigated numerically the electro-osmotic flow in a full capillary where the Joule heating effect and the thermal end effects are present using temperature dependent liquid properties. Their results show that the electric field intensity is non-uniform due to the reservoir-based thermal end effects. Xuan and Li [17] showed that Joule heating effects on the electric field, the temperature field and the flow field in a capillary should be considered simultaneously via temperature dependent electrical conductivity, thermal conductivity and viscosity of the liquid. The authors developed an analytical formulae for the electro-osmotic flow by considering only the radial temperature profile in the capillary without including the finite thermal conductivity at the capillary wall. Tang et al. [18] studied numerically the electro-osmotic flow in parallel plate microchannels by considering a power-law non-Newtonian

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