Oscillatory heat transfer process in a vertical strip immersed in a porous medium

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In this work, the transient conjugated heat transfer characteristics of a thin long vertical strip embedded in a porous medium has been studied using numerical and asymptotic techniques. The oscillatory nondimensional temperature evolution in the strip and the reduced Nusselt number at the top of the strip are obtained as a function of the Fourier number, Fo, which relates the fluctuating to the steady thermal penetration lengths. The numerical results of the root mean square of the fluctuating reduced Nusselt number have been obtained covering the complete transition from fast (Fo $\ll 1$) to slow (Fo $\gg 1$) thermal oscillations. Asymptotic solutions are obtained in these two limits and compared with the numerical results.

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I. INTRODUCTION

The natural convective conjugate heat transfer of a solid embedded in a porous medium represents a fundamental problem which has been investigated during the past decades. The excellent book by Pop and Ingham [1] presents abundant theoretical evidence of this thermal coupling interaction. In the same line, the book of Sundén and Heggs [2] shows that this simple geometric configuration is encountered in a broad range of scientific and engineering problems associated with different industrial applications. Additional examples and related topics can be found in the books by Ingham and Pop [3], Nield and Bejan [4] and Vafai [5]. The first theoretical study to treat the conjugate conduction-free convection problem of a long vertical thin strip or fin embedded in a porous media with a heated base was provided by Lock and Gunn [6]. Several works appeared in the past focusing mainly in obtaining self-similar and quasisimilarity solutions see for example the works of Cheng and Minkowycz [7–9]. Based on these studies, Pop et al. [10], obtained a set of similarity solutions for a long vertical plate projecting downward from a heated horizontal plane base at uniform temperature, for the case of the thermal conductivity-fin thickness product varying as a power function of distance from a certain specified origin. Later, Pop *et al.* [11], refining the above analysis, developed a finite-difference numerical scheme considering uniform the thermal conductivity and the thickness of the fin. Pop and Nakayama [12] made a modern review of the conjugate convective heat transfer from a vertical fin embedded in a fluid-saturated porous media. In the majority of the treated cases, the authors normally

consider an infinitely long fin to formulate the coupling governing equations. Recently, Martínez-Suastegui et al. [14] used numerical and asymptotic techniques to study the case of a finite length of a vertical strip embedded in a porous medium. The full transition from a short to a long strip has been studied.

In this work, the transfer heat transfer process in the limit of large strips is analyzed. The temperature at the top of the strip is assumed to fluctuate around a mean, which is higher than the ambient temperature in the porous medium far from the strip. For simplicity, the fluctuating temperature at the top of the strip is assumed to be harmonic with a given frequency. The corresponding steady state problem has been studied in [7, 10, 14], where the temperature decays algebraic towards the ambient temperature. The transient conjugate free convective boundary layer problem is found to depend on two nondimensional parameters. The Fourier number relates the transient to the steady penetration lengths. The second nondimensional parameter relates the amplitude of the thermal oscillations to the steady temperature difference.

II. BASIC EQUATIONS

The physical model and a suitable coordinate system are given in Fig.1. A vertical heated conducting strip of length L and thickness 2H, is totally embedded in a fluid-saturated porous medium with a temperature T_{∞} . The upper surface of the strip is assumed to be with a fluctuating temperature $T_0 + \Delta T \sin(\omega t)$, with $T_0 > T_{\infty}$ and $\Delta T/(T_0 - T_{\infty}) \ll 1$, whilst the lower surface is assumed to be adiabatic. Heat is transferred from the top of the strip to the fluid-saturated porous medium through the strip. Consider first an infinitely long strip. Owing to the heat loss to the surrounding fluid-saturated

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